Teaching mathematics through play
- an overview

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1. Introduction

Maths is perceived as boring and difficult by most of the children. This impression starts forming since the primary school days, and becomes difficult to overcome in the later stage of schooling. There is a need of making mathematics a fun experience akin to play. Thus, ‘Teaching mathematics through play’ was taken up as a topic to understand the alternate and interesting ways in which the subject of mathematics can be taught to the primary school children.

If we observe children closely, their play involves the same set of activities being done over and over again, which helps one ‘play’ better, but with out getting tired or bored. Learning involves a similar situation where problems are solved over and over again to be able to ‘solve’ better. With this similarity existing in both the activities, why can’t learning mathematics become more fun? more interesting?

This led me to investigate what can be done to make learning mathematics an enjoyable experience and what can I contribute as a designer.

2.1 play and mathematics

An interesting question that comes to mind, what is it in play that we want to bring in mathematics learning? While learning mathematics there is huge amount of resistance on the childs part which vanishes while playing. Through play, many things are learnt by the child which remains with him as tacit knowledge. Most importantly the child is highly receptive to understanding new things.

Teaching mathematics somehow restricts to the schools only as not all parents can come up with puzzles and riddles that can be played in the households. Generally schools have a very formal approach to learning, with very few exceptions. This leaves children devoid of any excitement in learning, thus they draw away from the experience of learning and understanding new concepts.

The children need to be highly receptive and actively involved in order to learn better. This demands improvement in teaching environment which needs to be more friendly and exciting. The teacher should be open to doubts and encourage children to vocalize their opinions. Understanding the psychology of children and ways of getting them involved in studies is an important training for all teachers.

Children always enjoy playing and can indulge in the activities for hours altogether. Many games the children play include a learning component, some don't. The children may play on their own or in groups, they may play witin a set of rules or just explore, there are many types of play. We would need to see what types of play would occur in a situation where learning component is a included.

2.2 types of play:

While teaching ‘mathematics through play’ an essential component that would come is that it can not be devoid of some learning component. Thus need comes of seeing what types of play can we identify with in our situation. Thus play can be classified as

1. Guided play- where the child (or a group of children) is guided through a game by a facilitator, directing him/her towards a specific goal.
2. Self motivated play- where the child explores a game, a concept on his own, without any participation of peers or guidance of an instructor.
3. Group play – where the child plays with his peers which helps in collective learning.

2.3 Concept formation:

A vital information we need to build a bridge between play and learning is understanding how children form mathematical concepts in their minds.

Mathematics is an abstract subject. It involves a hierarchy of abstractions. For example, 'two' is an abstract concept. You cannot understand two until you have met many pairs (a pair of eyes, a pair of shoes etc), and abstracted what all pairs have in common. You cannot nderstand what is meant by 'number' ntil you have understood 'two', 'three', 'four' and other similar concepts. 'number' is an abstraction from a set of abstractions. The concept of 'addition of numbers' is an even higher abstraction than 'number'. Thus we see that mathematics involves a hierarchy of abstractions, and we cannot understand any mathematical concept without also understanding the concepts on which it is depends lower in the hierarchy.

A child develops abstract thought by first experiencing the objects around him, then through spoken language that describes that experience, he would then start recognising pictures of the object. Much later he learns the symbol that we write to represent the sound 'ball'. This is sophesticated as the symbol has no properties at all in common with the real ball, and it is only artificially assosiated with the sounds that we utter in saying the word 'ball'.

Thus we realize that concept formation happens over a period of time, taking the earlier experiences into account and relating back to them, testing the assumptions in the given set of rules.

Four basic activities, introduced in pre-primary, form the base for concept formation in children. These are ‘matching’, ‘sorting’, ‘pairing’, and ‘ordering’. Matching is the way children learn to use language correctly- in particular, the language of mathematics. In any matching process, you are selecting experiences that have the property that you are seeking and discarding those that do not. For example, to have a concept of green, you must know what *is not* green as well as what *is* green. Sorting involves breaking up a set into new sets of matching individuals. Both, matching and sorting are activities that often arise in free play and in tidying after play. Pairing is an activity important for introducing numbers, ‘many’ and ‘few’ are imprecise terms that suggest comparison of contrasting numbers. More precise terms relating to numbers are ‘as many as’, ‘more than’ and ‘fewer than’. Ordering a set of objects by no particular rule invites the concepts of ‘first’, ‘next to’, ‘last’, ‘between’.

While designing activities for children, it is advisable to be aware of the possibility that children might be misled into selecting common properties other that those that truly identify the concepts under consideration. This is called ‘noise’ in psychological terms. To avoid misleading children by noise, we need to devise varied activities, with varied materials, to introduce any one concept. The key to successful mathematics teaching lies in repetition, varying activities as much as possible while maintaining the essential ingredient that invites the concept to be learnt. Such repetition guards against noise, acts as a memory aid and invites children to widen their horizons by applying the new concept to unfamiliar situations.

After understanding how concept formation happens in children, we need to understand what all is taught in the subject of mathematics in the primary classes.

2.4 primary mathematics:

What is taught to the primary school kids under the subject of mathematics? Each school follows different syllabus according to the board it is attached to; each state has its own state board, then there is CBSE and ICSE boards. NCERT develops books for the CBSE board, this syllabus was analyzed as it is used by most of the schools in the country. Various heads under which the syllabus is built are ‘geometry’, ‘numbers’, ‘money’, ’measurements’, ‘data handling’ and ‘patterns’. The fig 2.4(a)-(e) show the syllabus of class 1 to class 5.

The texts for class 1 to class 3 are beautifully illustrated but become a little too practical looking for class 4 and 5. For the class 1st to 3rd some of the principles are illustrated through games and puzzles, various patterns in numbers are discovered through puzzles. Games and puzzles form an essential part of generating a sense of play. A few exercises also require physical activities, the teachers help the students here, by acting as facilitators.

Active participation of students need to be achieved in order to get them interested in the subject of mathematics. Somewhere the pressure of completing the set syllabus in the given time of the year, takes away the freedom of the teacher to innovate exercises, conduct discussions on the taught topic. These should be some of the mandatory activities in the classroom to develop the capacity to articulating and reasoning in the children.

There have been different people working in the field of child education like Rudolf Steiner, J Krishnamoorthy, Montessori, Jean Piaget etc who have contributed immensely. But the proliferation of active learning has not happened so much in the Indian schools, specifically the state board schools. The state board schools or 'government schools' as they are called are the ones responsible for educating most of the Indian population. A few organizations like Navnirmiti in Mumbai have been working at this grass root level for the last few years now.

For the primary classes a lot of emphasis should be given to keeping their attention on the subject. This could be done by inclusion of activities like music, dance or drama in the classroom which aid in learning basic concepts of mathematics. Involving more than one of the five senses at a time in essential during the initial phases of learning. Making the children creative problem solvers is the need of the day. In the past, when knowledge was considered relatively fixed and limited, the most efficient education consisted of lecture, drill, and memorization. In an era of technological revolution and social change, the foundation of a good education is to learn how to learn.

Class I

Geometry (10 hrs.)
Shapes & Spatial Understanding

- Develops and uses vocabulary of spatial relationship (Top, Bottom, On, Under, Inside, Outside, Above, Below, Near, Far, Before, After)

Solids around us

- Collects objects from the surroundings having different sizes and shapes like pebbles, boxes, balls, cones, pipes, etc.
- Sorts, Classifies and describes the objects on the basis of shapes, and other observable properties.
- Observes and describes the way shapes affect movements like rolling and sliding.
- Sorts 2 - D shapes such as flat objects made of card etc.

Numbers
(46 hrs.)

Developing a sense of Numberness, Counting and Operations of Numbers 1 - 9 & zero

- Observes object and makes collections of objects.
- Arranges the collection of objects in order by
  - Matching and
  - One to one correspondence
- Counts the number of objects in a collection.
- Makes collection of objects corresponding to a specific number.
- Recognizes and speaks numbers from 1 to 9.
- Uses numbers from 1 to 9 in counting and comparison. (Real objects and repeated events like clapping to be used for counting)
- Reads and writes numerals from 1 to 9.
- Adds and subtracts using real objects and pictures.
- Adds and subtracts the numbers using symbols '+' and '-'.
- Approaches zero through the subtraction pattern (such as 3-1=2, 3-2=1, 3-3=0).

Numbers from (10 - 20)

- Forms Number sequence from 10 to 20.
- Counts objects using these numbers.
- Groups objects into a group of 10s and single objects.
- Develops the vocabulary of group of 'tens' and 'ones'.
- Shows the group of tens and ones by drawing.
- Counts the number of tens and ones in a given number.
- Writes the numerals for eleven to nineteen.
- Writes numerals for ten and twenty.
- Compares numbers upto 20.

Addition and Subtraction (upto 20)

- Adds and subtracts numbers upto 20.

Numbers from 21 - 99

- Writes numerals for Twenty-one to Ninety nine.
- Groups objects into tens and ones.
- Draws representation for groups of ten and ones.
- Groups a number orally into tens and ones.

Mental Arithmetic

- Adds two single digit numbers mentally.

Money
(3 hrs.)

- Identifies common currency notes and coins.
- Puts together small amounts of money.

Measurement
(13 hrs.)

Length

- Distinguishes between near, far, thin, thick, longer/taller, shorter, high, low.
- Seriates objects by comparing their length.
- Measures short lengths in terms of non-uniform units (in the context of games e.g. 'Gilli Danda' and 'marble-games').
- Estimates distance and length, and verifies using non-uniform units (e.g. hand span etc.)

Weight

- Compares between heavy and light objects.

Time

- Distinguishes between events occurring in time using terms - earlier and later.
- Gets the qualitative feel of long & short duration, of school days v/s holidays.
- Narrates the sequence of events in a day.

Data Handling
(6 hrs.)

- Collects, represents and interprets simple data such as measuring the arm length or circumference of the head using a paper strip.

Patterns
(10 hrs.)

- Describes sequences of simple patterns found in shapes in the surroundings and in numbers, e.g. stamping activity using fingers and thumb.
- Completes a given sequence of simple patterns found in shapes in the surroundings and in numbers.

Fig2.1 : NCERT syllabus for class I

Class II

Geometry (13 hrs.)
Shapes & Spatial Understanding

3-D and 2-D Shapes

- Observes objects in the environment and gets a qualitative feel for their geometrical attributes.
- Identifies the basic 3-D shapes such as cuboid, cylinder, cone, sphere by their names.
- Traces the 2-D outlines of 3-D objects.
- Observes and identifies these 2-D shapes.
- Identifies 2-D shapes viz., rectangle, square, triangle, circle by their names.
- Describes intuitively the properties of these 2-D shapes.
- Identifies and makes straight lines by folding, straight edged objects, stretched strings and draws free hand and with a ruler.
- Draws horizontal, vertical and slant lines (free hand).
- Distinguishes between straight and curved lines.
- Identifies objects by observing their shadows.

Numbers
(46 hrs.)

- Reads and writes numerals for numbers up to ninety-nine.
- Expands a number with respect to place values.
- Counts and regroups objects into tens and ones.
- Uses the concept of place value in the comparison of numbers.
- Counts in various ways:
  - Starting from any number.
  - Group counting etc.
- Arranges numbers upto hundred in ascending and descending order.
- Forms the greatest and the smallest two digit numbers with and without repetition of given digits.
- Indicates and identifies the position of an object in a line.

Addition and Subtraction

- Adds and subtracts two digit numbers by drawing representations of tens and ones without and with regrouping.
- Adds zero to a number and subtracts zero from a number.
- Observes the commutative property of addition through patterns.
- Solves addition, subtraction problems presented through pictures and verbal description.
- Describes orally the situations that correspond to the given addition and subtraction facts.
- Estimates the result of addition and subtraction and compares the result with another given number.

Preparation for Multiplication & Division

- Discussion of situations involving repeated addition and situations involving equal sharing.
- Activities of making equal groups.

Mental Arithmetic

- Adds and subtracts single digit numbers mentally.
- Adds and subtracts multiples of ten mentally

Money
(3 hrs.)

Money

- Identifies currency - notes and coins.
- Puts together amounts of money not exceeding Rs 50/-.
- Adds and subtracts small amounts of money mentally.
- Transacts an amount using 3-

Measurement
(13 hrs.)

Length

- Measures lengths & distances along short & long paths using uniform (non-standard) units, extends to longer lengths.

Weight

- Compares two or more objects by their weight.
- Appreciates the need for a simple balance.
- Compares weights of given objects using simple balance.

Capacity (Volume)

- Compares and orders containers in terms of internal volume(capacity).
- Orders given containers as per their capacities on the basis of perception & verifies by pouring out etc.

Time

- Gets familiar with the days of the week and months of the year.
- Gets a feel for sequence of seasons (varying locally).
- Sequences the events occurring over longer periods in terms of dates/days.

Data Handling
(6 hrs.)

- Collects data through measurement.
- Represents the data followed by discussion (e.g. heights of children).
- Collects and presents the data on birthdays.
- Draws inferences from the data at the appropriate level.

Patterns
(10 hrs.)

- Observes and extends patterns in sequence of shapes and numbers.
- Searches for patterns in different ways of splitting a number.
- Creates block patterns by stamping thumbprints, leaf prints, vegetable prints, etc.
- Creates patterns of regular shapes by stamping.

Fig 2.2 :NCERT syllabus for class II

Class III

Geometry (16 hrs.)
Shapes & Spatial Understanding

- Creates shapes through paper folding, paper cutting.
- Identifies 2-D shapes
- Describes the various 2-D shapes by counting their sides, corners and diagonals.
- Makes shapes on the dot-grid using straight lines and curves.
- Creates shapes using tangram pieces.
- Matches the properties of two 2-D shapes by observing their sides and corners (vertices).
- Tiles a given region using a tile of a given shape.
- Distinguishes between shapes that tile and that do not tile.
- Intuitive idea of a map. Reads simple maps (not necessarily scaled)
- Draws some 3D-objects.

Numbers
(42 hrs.)

Number sequence upto 1000

- Reads and writes 3-digit numbers.
- Expands a number w.r.t. place values.
- Counts in different ways - starting from any number.
- Compares numbers.
- Forms greatest and smallest numbers using given digits.

Addition & Subtraction

- Adds and subtracts numbers by writing them vertically in the following two cases -
  - without regrouping.
  - with regrouping.
- Uses the place value in standard algorithm of addition and subtraction.
- Solves addition and subtraction problems in different situations presented through pictures and stories.
- Frames problems for addition and subtraction facts.
- Estimates the sum of, and difference between, two given numbers.

Multiplication

- Explains the meaning of multiplication (as repeated addition).
- Identifies the sign of multiplication.
- Constructs the multiplication tables of 2,3,4,5 and 10
- Uses multiplication facts in situations.
- Multiplies two digit numbers using standard algorithm and Lattice multiplication algorithm.

Division:

- Explains the meaning of division from context of equal grouping and sharing.
- Relates division with multiplication.
- Completes division facts
  - by grouping
  - by using multiplication tables.

Mental Arithmetic

- Adds and subtracts single digit numbers and two digit numbers mentally.
- Doubles two digit numbers mentally (result not exceeding two digits).

Money
(5 hrs.)

Money

- Converts Rupee. to Paise using play money.
- Adds and subtracts amounts using column addition, and subtraction without regrouping.
- Makes rate charts and bills.

Measurement
(21 hrs.)

Length

- Appreciates the need for a standard unit.
- Measures length using appropriate standard units of length by choosing between centimetres. and metres.
- Estimates the length of given object in standard units and verifies by measuring.
- Uses a ruler
- Relates centimetre. and metre.

Weight

- Weighs objects using non standard Units.
- Appreciates the conservation of weight.

Volume

- Measures and compares the capacity of different containers in terms of non-standard units.
- Appreciates the conservation of volume.

Time

- Reads a calendar to find a particular day and date.
- Reads the time correct to the hour.
- Sequences the events chronologically.

Data Handling
(6 hrs.)

- Records data using tally marks.
- Collects data and represents in terms of pictograph choosing appropriate scale and unit for display through pictographs.
- Draws conclusions from the data by discussing with the teacher.

Patterns
(6 hrs.)

- Identifies simple symmetrical shapes and patterns.
- Makes patterns and designs from straight lines and other geometrical shapes.
- Identifies patterns in the numerals for odd and even numbers and in adding odd and even numbers.
- Partitions a number in different ways.
- Identifies patterns in his surroundings
- Identifies patterns in multiplication with, and dividing by 10s.

Fig 2.3 :NCERT syllabus for class III

Class IV

Geometry (16 hrs.)
Shapes & Spatial Understanding

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- Draws a circle free hand and with compass.
- Identifies centre, radius and diameter of a circle.
- Uses Tangrams to create different shapes.
- Tiles geometrical shapes: using one or two shapes.
- Chooses a tile among a given number of tiles that can tile a given region both intuitively and experimentally.
- Explores intuitively the area and perimeter of simple shapes.
- Makes 4-faced, 5-faced and 6-faced cubes from given nets especially designed for the same.
- Explores intuitively the reflections through inkblots, paper cutting and paper folding.
- Reads and draws 3-D objects, making use of the familiarity with the conventions used in this.
- Draws intuitively the plan, elevation and side view of simple objects.

Numbers (40 hrs.)

Numbers and operations

- Writes multiplication facts.
- Writes tables upto 10x10.
- Multiplies two and three digit numbers using lattice algorithm and the standard (column) algorithm.
- Divides a given number by another number in various ways such as:
  - by drawing dots.
  - by grouping.
  - by using multiplication facts.
  - by repeated subtraction.
- Applies the four operations to life situations.
- Frames word problems.
- Estimates sums, differences and products of given numbers.

Mental Arithmetic

- Adds and subtracts multiples of 10 and 100, mentally.
- Completes multiplication facts by adding partial products, mentally (e.g. 7x6 = 5x6+2x6).

FRACTIONAL NUMBERS

- Identifies half, one fourth and three-fourths of a whole.
- Identifies the symbols, ½, ¼, ¾.
- Explains the meaning of ½ , ¼ and ¾ .
- Appreciates equivalence of 2/4 and 1/2; and of 2/2, 3/3, 4/4 and 1.

Money (5 hrs.)

Money

- Converts Rupees to Paise.
- Adds and subtracts amounts using column addition and subtraction with regrouping.
- Uses operations to find totals, change, multiple costs and unit cost.
- Estimates roughly the totals and total cost.

Measurement (21 hrs.)

Length

- Relates metre with centimetre;
- Converts metre into centimetres and vice versa.
- Solves problems involving length and distances.
- Estimates length of an object and distance between two given locations.

Weight

- Weighs objects using a balance and standard units.
- Determines sums and differences of weights.
- Estimates the weight of an object and verifies using a balance.

Volume

- Measures volumes of given liquid using containers marked with standard units.
- Determines sums and differences of volumes.
- Estimates the volume of a liquid contained in a vessel and verifies by measuring.

Time

- Computes the number of weeks in a year.
- Correlates the number of days in a year with the number of days in each month.
- Justifies the reason for the need of a leap year.
- Reads clock time to the nearest hours and minutes.
- Expresses time, using the terms, 'a.m.' and 'p.m.'
- Estimates the duration of familiar events.
- Finds approximate time elapsed by (to the nearest hour) forward counting.
- Computes the number of days between two dates.

Data Handling (6 hrs.)

- Collects data and represents in the form of bar graphs;
- Draws Inferences by discussing with the teacher.

Patterns (6 hrs.)

- Identifies patterns in multiplication and division: multiples of 9,
- Casts out nines from a given number to check if it is a multiple of nine.
- Multiplies and divides by 10s, 100s.
- Identifies geometrical patterns based on symmetry.

Fig 2.4: NCERT syllabus for class IV

Class V

Geometry (hrs.)
Shapes & Spatial Understanding

- Gets the feel of perspective while drawing a 3-D object in 2-D.
- Gets the feel of an angle through observation and paper folding.
- Identifies right angles in the environment.
- Classifies angles into right, acute and obtuse angles.
- Represents right angle, acute angle and obtuse angle by drawing and tracing.
- Explores intuitively rotations and reflections of familiar 2-D shapes.
- Explores intuitively symmetry in familiar 3-D shapes.
- Makes the shapes of cubes, cylinders and cones using nets especially designed for this purpose.

Numbers (40 hrs.)

Numbers and operations

- Finds place value in numbers beyond 1000.
- Appreciates the role of place value in addition, subtraction and multiplication algorithms.
- Uses informal and standard division algorithms.
- Explains the meaning of factors and multiples.

Mental Arithmetic

- Estimates sums, differences, products and quotients and verifies using approximation.

Fractional Numbers

- Finds the fractional part of a collection.
- Compares fractions.
- Identifies equivalent fractions.
- Estimates the degree of closeness of a fraction to known fractions (½, ¼, ¾ etc.)
- Uses decimal fractions in the context of units of length and money.
- Expresses a given fraction in decimal notation and vice versa.

Money (5 hrs.)

- Applies the four operations in solving problems involving money.

Measurement (26 hrs.)

- Determines area and perimeter of simple geometrical figures.
- Applies the four operations in solving problems involving length, weight and volume.
- Relates commonly used larger and smaller units of length, weight and volume and converts one to the other.
- Applies simple fractions to quantities.
- Converts fractional larger unit into complete smaller units.
- Appreciates volume of a solid body: intuitively and also by informal measurement.
- Uses addition and subtraction in finding time intervals in simple cases.

Data Handling (6 hrs.)

- Collects two-dimensional quantitative data.
- represents the data in the form of a table.
- Draws a bar graph or a pictograph to present a data.

Patterns (6 hrs.)

- Identifies patterns in square numbers, triangular numbers.
- Relates sequences of odd numbers between consecutive square numbers.
- Makes border strip and tiling patterns.

Fig 2.5: NCERT syllabus for class V

3.1 Theories Of Learning

3.1.1 Piaget’s Theory Of Learning

3.1.2 Bruner's Theory

3.1.3 Skemp’s Theory

3.1.4 Dienes Theory

3.1.5 Conclusions

3.1 THEORIES OF LEARNING

Various theories have been generated by the leading psychologists for learning. Here, four theories of learning are given which have been constructed by four psychologists whose research are mainly concerned with children's mathematical learning.

3.1.1. Piaget's Theory Of Learning

Piaget postulated three basic learning processes. They are

Formation: formation of mental concepts. For example a baby shows that he has a concept of 'ball' when he retrieves his ball from underneath a blanket placed over it. His senses did not tell him that the ball was there, his mind did.

Adaptation: adaptation of these concepts in the light of experience the child shows that he has adapted the concept of ball when he calls other objects ball. Ball for him means not just his own ball, but a class of objects that match his ball in some specific way.

Relation: relating of concepts to form structures. The child shows that he has related concepts to form a structure when he tells that ball 'rolls'. The sentence relates the two concepts ball and rolls

According to Piaget Adaptation is most important as he claims that adaptation is the essential ingredient of learning. He says humans adapt in one of two ways.

Assimilation: a process of fitting new experiences to the existing concepts. For example a child who has seen five cars and when describes five kittens is assimilating a new experience into his concept of five.

Accommodation: a correcting process by which one either restricts or broadens the concepts. For example children who have learnt that symbol '5' represents 'Five' have to broaden their concept of this symbol when they are faced with the numeral 54. now the symbol '5' represents 'Fifty'.

3.1.2. Skemp's 'Theory Of Learning'

British psychologist Richard Skemp introduced the idea that concepts constructed by humans form hierarchy. For example the concept of 'red' is primary concept, because it depends exclusively on data from our senses (sight). 'Colour' he says is a secondary concept because one cannot form the concept of colour until one recognizes that objects may exhibit the sense data of red, yellow, blue and so on. 'two' is also a secondary concept because it depends upon the recognition of primary concepts. The hierarchy of concepts continues to build up. One cannot form a concept of number until one has formed the concepts of two, three, four and so on. So number is a tertiary concept. Addition is fourth degree concept. According to Skemp, mathematics, involves an extensive hierarchy of concepts.

Skemp has proposed a 'theory of learning' which takes into account that emotions play a dominant part in the way one learns. In this theory he states that there is a 'director system' in each and every organism which directs and organizes its behaviour towards a GOAL.

The function of director system is governed by emotions, such as,

Pleasure- which signals the approach or reaching of a goal state

Confidence- signals the ability to reach a goal state.

Displeasure- which signals a retreat from a goal state.

Frustration- which signals the inability to approach or reach a goal state.

As the goal states, there are anti-goal states too. Emotions attached to these are,

Fear – which signals their approach

Anxiety- which signals inability to avoid them

Relief- which signals a retreat from them

Security- which signals the ability to retreat from them

For mathematical problems, the director system registers it and passes the message to the emotional system. The emotional system feeds back the message of either goal states or anti-goal states.

3.1.3. Bruner's Theory Of Instruction

Jerome Bruner is one psychologist who has challenged Piaget's theory of learning that learning is completely subordinate to biological development. In contrast to Piaget, Bruner has claimed that 'any idea or body of knowledge can be presented in a form simple enough so that any particular learner can understand it in recognizable form'. This idea seems to be something of an overstatement as the effort of Bruner to teach four year old conservation of liquids appeared to have no effect on them.

Bruner has constructed a theory of learning that is particularly interesting in the context of mathematics. According to him learning consists essentially of concept formation, which is 'the multiple embodiment of abstract idea in different physical forms'. We need a store of mental images on which to draw in order to form these abstract ideas; our means of building concepts, he says, lie in the three modes of representing the world : (a) the enactive mode (b) the iconic mode and (c) the symbolic mode

Bruner suggests that the essential points for teachers to consider are the following:

Children's predisposition towards learning: children are predisposed to learning because they have an intrinsic curiosity. This curiosity has to be properly channeled into what he calls as 'guided discovery' which comes from carefully selected experience and its representation in language, pictures and symbols.

Structure of knowledge: he emphasizes that exercises that children are asked to perform should 'cry out' for simplification into conceptual form. For example, the addition problems like 3+2=… 2+3=… and 2+4= … 4+2 =…. cry out for the concept that addition is commutative. The teachers role, says Bruner, is to help children verbalize what they have done, so that they can develop the required concepts.

Sequence: according to him there is no best sequence for mathematics. There are different paths to the same goal and because of individual differences it is important to build several. He asserts that one needs a 'spiral curriculum'. At first the ideas are presented in language that is imprecise but honest. At a later stage the same ideas must be revised and described more precisely. For example, the word 'square' will first be used by children to describe a primary concept, later they will associate it to the existence of equal lengths right angle, symmetry, four sides etc.

Motivation and reward: it is important, Bruner says, for children to feel that their work is leading towards a goal.the knowledge that they gain must be seen as a useful tool. The teacher should motivate them by them an idea of what they will be able to achieve with their knowledge. The most important reward for learning is not praise from an adult, but the intrinsic satisfaction

3.1.4. Dienes 'Theory Of Learning'

Zoltan Diene's theory of learning says that learning is a process of an increasingly intricate play. He says that these plays are of two types- primary play and secondary play. Mathematical play falls under these two categories.

Primary play is activity with materials aimed at gratifying immediate desires or instincts. It involves the manipulation and investigation of materials for its own sake.

Secondary play is activity performed with awareness, aimed at an end which is beyond the immediate gratification of desires. This involves trying to build with the materials, discovering patterns or irregularities and forming abstract conjectures or 'rules' concerning the patterns found.

Secondary play according to Dienes involves.

ABSTARCTION: is the process of extracting what is common to a number of different situations and discarding 'noise'. For example to label a given pair of objects 'two' requires the recall that other pair of objects were labeled 'two'.

SYMBOLIZATION: spoken and written symbols are used to represent classes which have been assembled by the process of abstraction. For a child who has formed the abstraction of two, three and five can be encouraged to express his discovery in written symbols. But Dienes says that is symbols are introduced too early, they can become an empty shell of sign with rules for manipulation, instead of an aid to thought.

GENERALIZATION: is the process of extending a class to include new situations for example children who predict that two things, and three more things will be five things, might be said to have generalized their experiences with two toys and three toys, two sweets and three sweets.

3.1.5. Conclusions

After going through all the above theories we may ask why then do children ‘fail’ at mathematics. A few things like the rate of learning for a child, the level of anxiety, understanding and attitude of the child act as key factors here.

A fanatic Piaget follower may claim that we can teach nothing to the children as their cognitive growth progresses at a predetermined rate, whereas a fanatic Bruner follower would claim that we can teach anything to the children. The truth lies somewhere in between these extremes, and is never same for two different children. The children not only learn in different ways but also at different rates.

We have seen the emphasis laid on ‘emotion’ and ‘motivtion’ by some of the theories in the earlier section. It is possible to get over motivated and anxious. Many psychologists have shown that high anxiety impedes learning. That is why play is so important. Children play not to become agile but because of the intrinsic pleasure that it gives them. They will learn mathematics best if they consider it as something pleasurable for its own sake.

Understanding is a continuing process. The ability to understand depends on ones ability to adapt, as Piaget would put it or ones ability to tolerate variability, as Dienes would put it.

Attitude is also an important factor in learning mathematics. Research shows that a child's attitude to mathematics seems to be consolidated by the age of eleven. Thus, adults who say, 'I cant do maths', are usually found to have formed this opinion by the age of eleven. If you don't like something, you tend to avoid it and perhaps fear it. If only we can keep children's attitude to mathematics positive and inqusitive up to the age of eleven, regardless of their ability, we might prevent the 'blockage' that is so often formed to resist the pain of learning any further mathematics.

Thus we see that emotion holds a great importance in mathematics learning. Children need emotional stimulus of real materials and problems in which they feel interest and involvement. They need language to talk about and analyze these problems. They also need picture and diagrams to clarify the essentials of a problem at its outset. Finally they need meaningful written symbols with which to solve and generalize a problem. Thus we see a great emphasis on experiential leaning and element of play as an old Chinese proverb says 'I hear and I forget,/ I see and I remember, / I do and I understand'.

4.1 VARIOUS ORGANIZATIONS WORKING IN THE FIELD

The need of introducing activity based learning is understood by many now. A few groups are working in the zone of developing activity based structures for learning mathematics in India. The platform that these organizations are working from are varied, some are working in collaboration with the school structure where as some are working independently. International perspective for primary education is also looked at.

I put forward a brief review of a few of these organizations and their plethora of work, in order to understand how much of activity and play element exists in the

4.1.1 Navnirmiti- Collaboration With School Structure

The organization started by Dr. Vivek Monterio and Geeta Mahashabde is an autonomous organization, situated in Mumbai. They believe that there is another world of mathematics, which is possible and accessible to all. Comprehensive structured system for teaching primary school level mathematics skills to every student using reality-based content and activity-based, dialogue methods and 'thing symbolic' approach.

Universal Active Mathematics Method (UAM Method): This method of teaching is for universalization of mathematics education. It uses the universal language of mathematics, that is the things-language. It uses Reality based content and activity-based Do and Discover method. It is aimed at equipping the students with a confident understanding of maths competencies. UAM method connects the real life math with its **things-language** representation as well as alphanumeric expression. This method is tried and tested in all types of schools - rural, tribal, local government schools in urban areas and even the elite schools in Mumbai. In all these schools the teacher-student ratio is 1 : 60 to 1 : 40.

In this situation it is extremely important to strengthen government school system and provide good quality education in these schools along with strong academic support to the teachers.

Steps for achieving this are as follows:

1. Developing a pedagogy appropriate for these children, with continuous experimentation, feedback and modifications and achieving the best that works in reality.
2. Imparting rigorous intensive training to the teachers and inculcating the culture of activity-based do-and-discover and dialogue methods.
3. Developing systems that work the best in these schools. This will include continuous support and monitoring.
4. Scientific evaluation of the entire process.

The organization works closely with schools and involves in teacher training workshops. Creating maths labs in the interested schools etc.

Training workshops: UAM – Introductory workshop: For primary school teachers. The workshop covers basic concepts of the primary school curriculum. It is an introductory workshop to 'Universal Active Mathematics'. It gives hands on experience of the activity-based, do & discover method of teaching and learning. Participants explore creative ways of math learning with the innovative math kit developed by Navnirmiti.

Math lab: A Math Lab is a space designed for students to learn mathematics by performing activities and hands on experience. A ' MATH LAB ' provides wide variety of materials to play with and learn the concepts. Math's teaching and learning for primary school level (i.e for class 1 to 4) can entirely be done with activities and experiments by students. Math Lab also encourages group learning and co-operative learning among children.

Website: www.navnirmiti.com

4.1.2 Azim Premji Foundation- Self Standing Teaching Platform

Azim premji foundation is a organization working with education. They focus on software based learning through the mobile learning bus, where the children who can not access school, like the children of migratory labour, come and play. These games are developed around the basic educational material in mind. Various subjects like maths, science, geography etc are taught through this platform.

The method uses content development faculty of the organization to create software based content but it lacks the linking required to physical activities. It also doesn't have much involvement with schools. Being in the game format there is not a system of assessment. Though it would be exciting if one needs to clear the levels with increasing difficulty to reach higher levels in the game. This will make sure that the child goes through the complete hierarchy of concepts without having to worry about the assessment. But a lot of different situations need to be put forward to the children to ensure they are learning the concepts without any 'noise'. A very important thing that this venture does is creating excitement about learning in the children which is of much value.

Website : www.azimpremjifoundation.com

4.1.3 The Pomgranate Workshop- Extra Curricular Mode

The group started by Priya Srinivasan and Lavanya Varadrajan to trigger creativity amongst children through various structured modules that layer multiple disciplines and focus on self expression and experiential learning in children.

This is mainly an informal learning platform which focuses on experiential learning and nurturing creativity amongst children. The subjects dealt with are not the formal school subjects, but more of creative disciplines. Some basic mathematical concepts were introduced through a few sessions to the children in a very playful setup. I present the analysis of one such session in the next section.

A very important factor in these workshops is that there is no formal assessment of the activities. High element of play and fun keeps the children engaged and receptive to the content delivered. The play is mostly guided but leads to explorations and discoveries of concepts by the students on their own. Discussions amongst the students help in the process of learning tremendously.

Website : www.thepomgranateworkshop.com

4.1.4 International Scenario – Teachers resources by Annenberg media

National Council of Teachers of Mathematics (NCTM) is the world's largest organization dedicated to improving teaching and learning of mathematics from pre-kindergarten to high school. It is a non-profit, nonpartisan education association affiliated to schools in united states and Canada. The NCTM board could be seen as a parallel to our NCERT or ICSE board here in India. A lot of free online material is available for children as well as teachers to learn from. This includes lessons for children, audio visual recordings of class lectures for teachers. A lot of audiovisual data is made available over internet with the help of Annenberg Media which has shot many classroom sessions of primary mathematics.

A huge emphasis is on motivating the teachers to become more innovative in their approach. There are videos of various classroom lessons for primary school children, which showcase the innovative approach of teachers and amount of freedom given to the students for expression and play.

Website http://learners.org/resources/series32.html

4.2 MULTIPLICATION

Multiplication is a number operation which comes under the realm of basic algebra. The topic is hardly visual and deals more with memorizing facts than visualizing them.

4.2.1 Introduction To Multiplication

Multiplication is introduced to children in the 3rd standard of their schooling life, i.e. at the age of approximately 7 years. According to Jean Piaget's theory this is the age where they can understand abstractions. Thus, this is the ideal age for the introduction of this topic.

4.2.2 Multiplication By Navnirmiti

A brief review of multiplication as taught by Navnirmiti is done here. Fig 4.1- Fig 4.5 show the process followed by Navnirmiti to teach multiplication.

The introduction to multiplication is done through various objects used in daily life like piles of lemon etc which help the children understand repetitive addition of equal groups. The children also use Jodo blocks and pegs they have used earlier while learning numbers.

The method then uses Dienes cubes to create experiential learning. The numbers are not written in symbolic language but are actually dealt with physically by moving from one place to other. Relationship between an unit to a tens value is easily experienced here.

Fig 4: Dienes blocks for units, tens, hundreds and thousand

UNIVERSAL ACTIVE MATH
Multiplication
A Systematic Structured Method
Based On the Principle that in Things Language

Every Multiplication gives a rectangle. Every rectangle is a multiplication.

Developed by Dr. Vivek Monteiro & Geeta Mahashabde

Important points about Universal Active Math

1. Learning by understanding and learning only by understanding.
2. A systematic pedagogy based on the Chinese proverb :
   " I hear, I forget; I see, I remember; I do, I understand"
3. Things language understanding first; then the alphanumeric language.
4. Systematic translation from things language to alphanumeric language.
5. Support, monitoring, assessment systems to ensure that every teacher and every child becomes comfortable with mathematics.

MULTIPLICATION IN THINGS LANGUAGE :

Use Jodo blocks / marbles / pebbles for the following sequence :

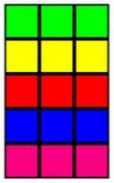
* Give me 2 + 3.
  With Jodo Blocks : Children will make : 
  With Pebbles : Children will give : 

  2 pebbles of one colour and 3 of other colour.
  OR 2 in one hand and 3 in the other hand and then joined together.

* Give me 3 + 3.

* Give me 3 + 3 + 3 + 3 + 3

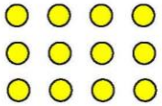
* When the same number is added (joined) to itself repeatedly we can also join the groups of 3 blocks differently, one below the other to make a rectangle. 3+3+3+3+3



* Bhajiwala has made piles of 4 lemons each.

This also has a multiplication :
Three times four : 3 x 4

The lemons can also be arranged thus to make a rectangle :

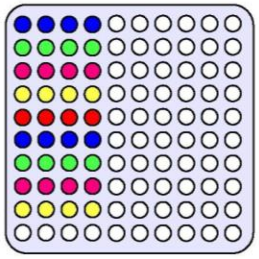


THUS WE CAN SAY THAT
EVERY MULTIPLICATION GIVES A RECTANGLE
AND
EVERY RECTANGLE IS A MULTIPLICATION.

Use mathemat and Jodo Blocks to make rectangles to represent multiplication.

5 x 4

9 x 4



All multiplication tables upto
10 x 10 can be made on the mathemat.

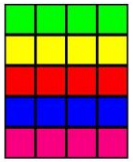
Fig 4.1: Navnirmiti multiplication lessons

When children learn multiplication as making rectangles they learn to connect :

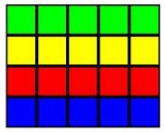
- multiplication with the associated rectangle.
- the number with its factors.
- the rectangular shape with the number.
- repeated addition and multiplication to the rectangle.
- multiplication with area

THE LINGUISTIC PROBLEM IN MULTIPLICATION

Pay attention to the linguistic problem!
5 x 4 is read in English as 'five times four' and indicates five groups of four each.



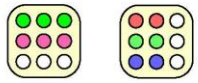
But 5 x 4 is read in Marathi or Hindi as 'paanch guna char' which means four groups of five each.



This write up uses the English language interpretation.

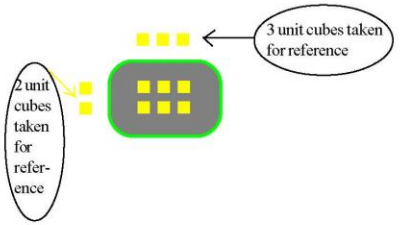
* Ask children to make 2 x 3 (Two times three. Two groups of three each).
* Ask them to make 3 x 2 (Three times two. Three groups of two each).

Children can verify for themselves that these are equal :



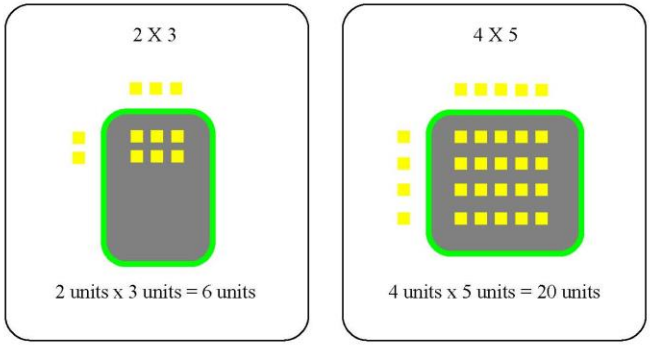
USE DIENES' BLOCKS (UTH kit) FOR FURTHER MULTIPLICATIONS.

2 x 3 : Two times three. Three taken two times



Children make a rectangle of 2 x 3 on their slate placing unit cubes as above.

UNITS TIMES UNITS GIVES UNITS



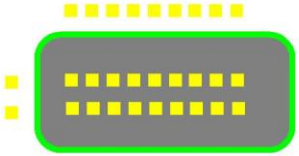
U X U = U

Children make rectangles with dienes' blocks and discover the rule U x U = U
(Of course, subsequently the 20 unit cubes can be joined to make 2 tens)
Note : While solving these problems children are not writing multiplication in alphanumeric language. They are making shapes and recognizing (counting) the result.

Fig 4.2: Navnirmiti multiplication lessons

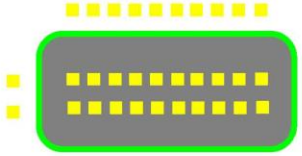
UNITS TIMES TENS IS TENS

2 x 9



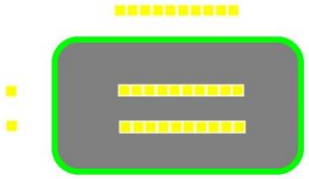
First ask children to make 2 x 9.
They will make this rectangle.

Then ask them to make 2 x 10- two times ten, or ten taken twice.
They may add one cube in each line to make this rectangle.

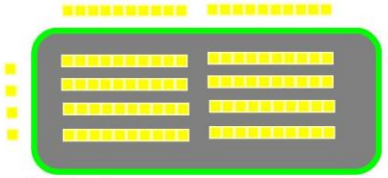


Children know that when they get ten cubes they can make a rod by joining them. Therefore in the above arrangement of 2 x 10, children join the cubes horizontally to make a rod. If they join the reference cubes to make a rod, they must also do the same to the 'answer' cubes.

2 x (1 ten)
Answer: Two tens



4 x (2 tens)
Answer : 8 tens

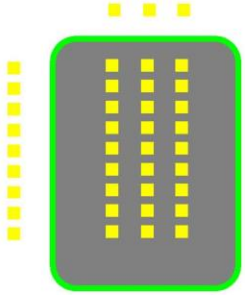


This way children discover the rule of

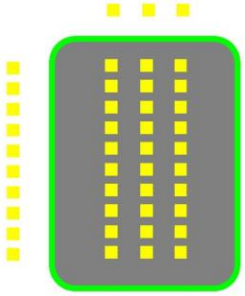
UNITS x TENS = TENS

TENS TIMES UNITS IS TENS

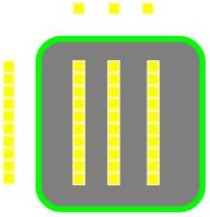
First ask 9 x 3.
Children will make this rectangle.



Then ask 10 x 3
They will add one cube to each column and will make this one.

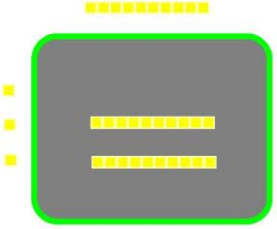


Children know that when they get ten cubes they can make a rod by joining them. Therefore in the above arrangement of 10 x 3, children join the cubes vertically to make rods.



This is how children discover the rule

TENS x UNITS = TENS

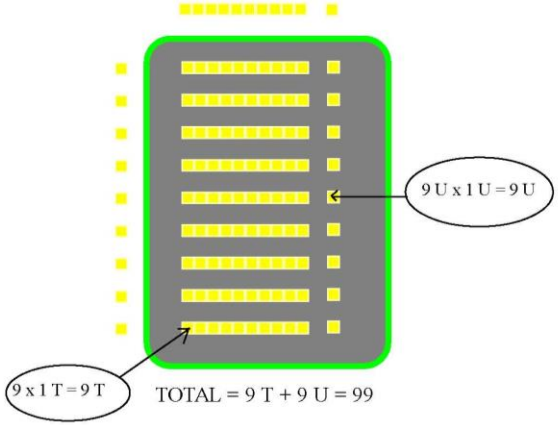


Ask them to compare (1 ten x 3 units) with (3 units x 1 ten) . Let them discuss among themselves the similarities and differnences.

Fig 4.3: Navnirmiti multiplication lessons

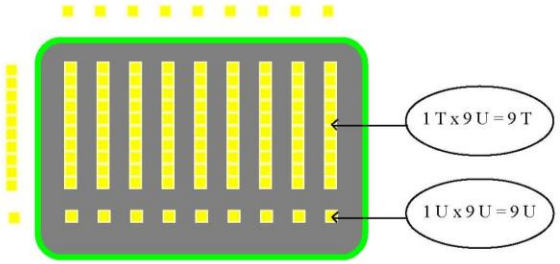
USING THE DISCOVERED RULES TOGETHER

* What is 9 x 11?
It is 9 times (10 + 1)
Which is 9 times a ten and nine times a unit.



TOTAL = 9 T + 9 U = 99

* What is 11 x 9?
It is nine units taken ten times, and then once more.
It is a ten times nine and a units times nine.

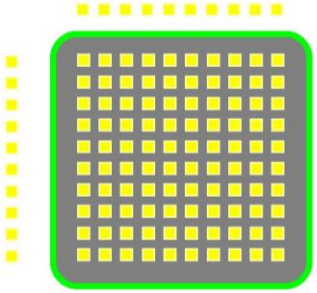


TOTAL = 9 T + 9 U = 99

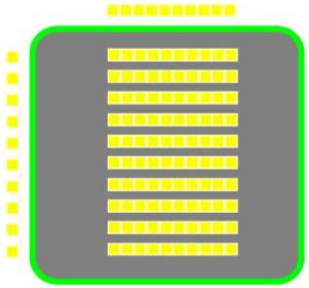
TENS TIMES TENS ARE HUNDREDS

Ask children 'what is 10 x 10?'
Children will do it in different ways. Each of the different ways can then be joined to make a hundred plate.

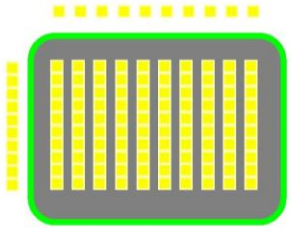
10 units x 10 units



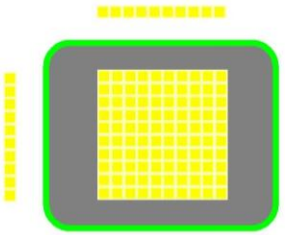
10 units x 1 ten



1 ten x 10 units



1 ten x 1 ten



By making squares of 10 x 10 in different ways, comparing each other's ways and discussing children discover the rule :

T x T = H

Now children have discovered four important rules :

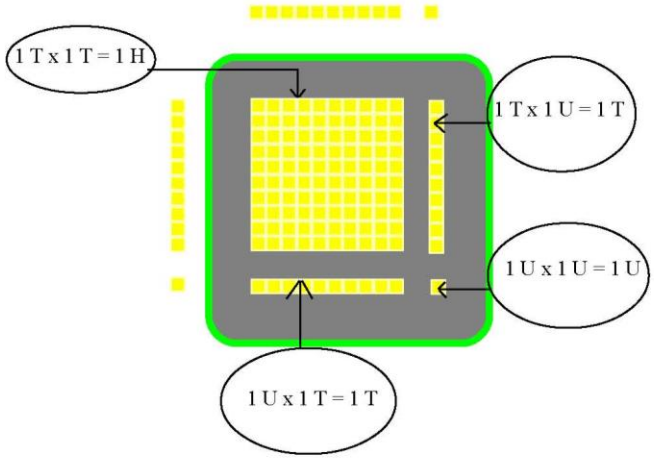
U x U = U
U x T = T
T x U = T
T x T = H

Fig 4.4: Navnirmiti multiplication lessons

Two Digit x Two digit

This understanding is now applied to solve a problem like 11 x 11 (two digit x two digit)

11 x 11 :

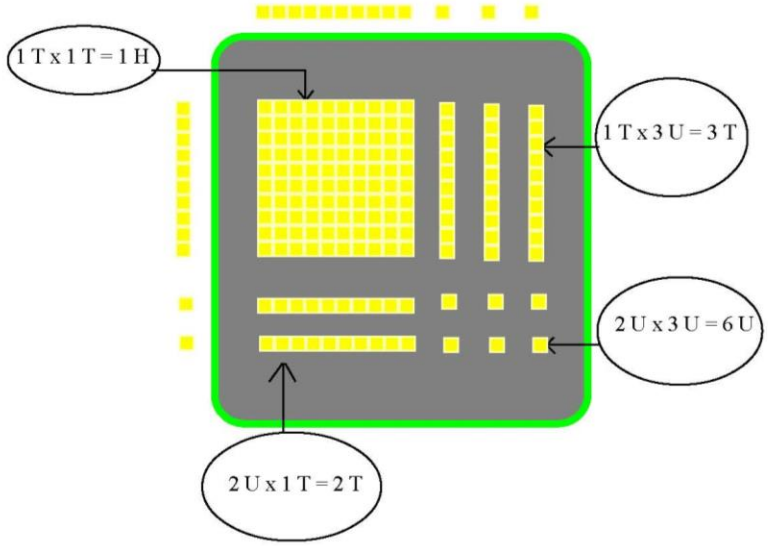


After 10 x 10, only ask what is 11 times 11. Children will struggle, will try out and will discover this square. Here the child has discovered that

$$11 \times 11 = (10 + 1) \times (10 + 1)$$
$$= (10 \times 10) + (10 \times 1) + (1 \times 10) + (1 \times 1)$$

We should not teach or mention this as a rule at this age. We should only give them the problem and when they make this square they have discovered it.

* 12 x 13

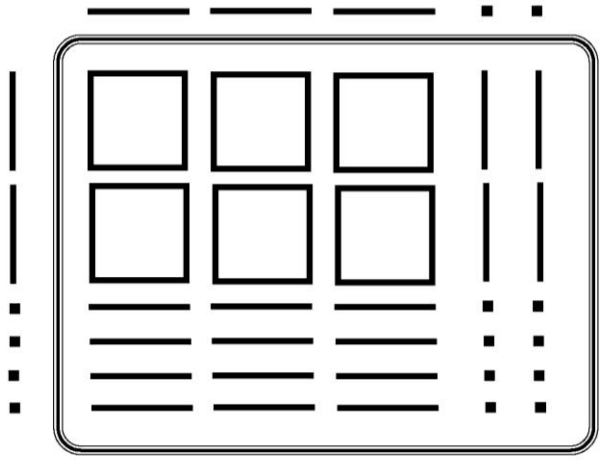


When children become comfortable in making rectangles and getting answers they have understood multiplication in 'Things Language'. This understanding is then translated into pictorial language where they represent multiplication using pictures. Then it is systematically translated into an alphanumeric metric represntation and then finally into the standard representation.

Fig 4.4: Navnirmiti multiplication lessons

Systematic translation into Pictorial Language :

* 24 x 32
  Children first make rectangle using plates, rods and cubes (UTH kit or Dienes' blocks) . Referring to the arrangement they draw picture as follows :



* The above picture is then represented as alphanumeric matrix :

	3 T	2 U
2 T	6 H	4 T
4 U	12 T	8 U

* From the matrix they translate it to the following grid in order to count the total number of hundreds, tens and units.

	H	T	U
	6		
		4	
+	1	2	
			8
	7	6	8

SWITCHING OVER TO STANDARD WAY OF WRITING MULTIPLICATION
Children have discovered the rules. That understanding is used to arrive at this.

	H	T	U	
		3	2	
×		2	4	
			8	2 units taken 4 times gives 8 units.
	1	2		3 tens taken 4 times gives 12 tens which is equal to 1 hundred and 2 units.
		4		2 units taken 2 T times is 4 tens.
+	6			3 tens taken 2 T times gives 6 hundreds.
	7	6	8	All added together give 768.

Fig 4.5: Navnirmiti multiplication lessons

4.2.3 Analysis

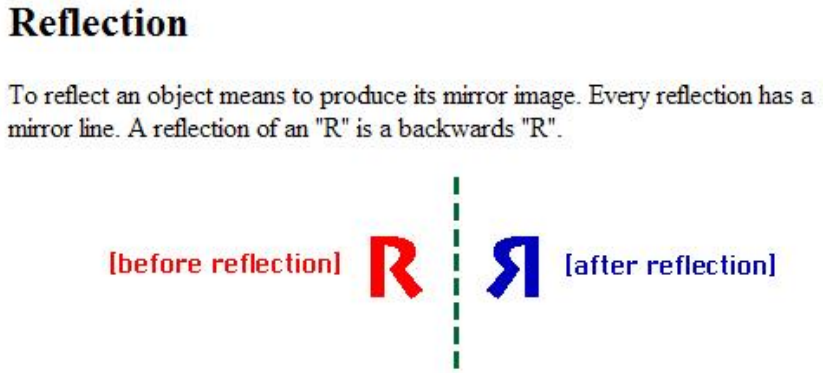
The attempt by Navnirmiti is commendable with the efforts being put in to incorporate a more activity based learning mode in the regular government schools. But the method lacks on few fronts when we check it for the elements of play. It was noticed that children still perceive the kit as a study material and not much like a game or play thing.

This is a side effect of an effort being made for validating the use of the kits which creates a compulsion of sticking to the textbooks of state boards and the same exam structure. The kits and methods of teaching are being evaluated scientifically and huge emphasis in on use of low cost materials. This would facilitate the implementation of the process on a wide scale, in both rural and urban scene.

But a very important aspect being taken care of by Navnirmiti is the teacher training sessions where the government school teachers get a chance to voice their opinions and understand the importance of teaching students through experiences and play.

4.3 SYMMETRY

Symmetry is a visual concept which is different from arithmetic which needs development of visual relation. Though, symmetry is so obvious and taken for granted that emphasizing its principles and putting them into words becomes important. Teaching mathematics to children is all about developing abstractions from the obvious and making the abstract concepts visible in the ordinary life. The journey needs to happen both ways.



4.3.1 introduction to symmetry

The symmetry introduced to children is reflection symmetry or mirror symmetry. The level of abstraction required for the other types of symmetry is much higher and would not be understood easily by the primary school kids.

In the age group of seven to nine years children start relating to abstract concepts. Bilateral symmetry or mirror symmetry is a concept that can be easily understood in this age. Other types of symmetries can be introduced to them but without expecting them to decode the principles.

4.3.2 Symmetry In A Workshop

The concept of symmetry was introduced to children of the age group 7-9 yrs. The platform to reach them was the pomegranates workshops, which offers a very informal environment for learning. Perceived more as a fun time by the children. Priya Srinivasan from 'The Pomgranate Workshop' was the coordinator for the session. The workshop was conducted by Prof. A. G. Rao assisted by Kiran Kulkarni and Rupesh Nath.

Various activities done to understand symmetry were

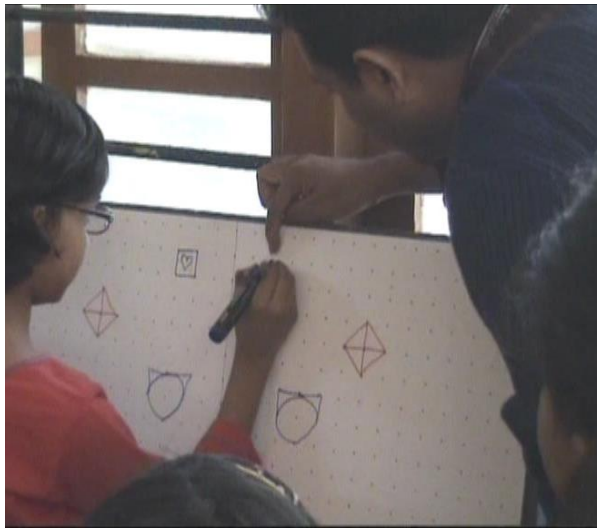
1. Definition of symmetry
2. symmetry by examples in nature
3. symmetry by hand gestures
4. symmetry by paper folding and cutting
5. symmetry by ink blotting
6. symmetry on a grid paper
7. Symmetry through acting
8. Symmetry around you
9. Understanding symmetry in shapes
10. Line of symmetry
11. Symmetry in alphabets
12. mirror manipulation-grid paper
13. mirror manipulation-words

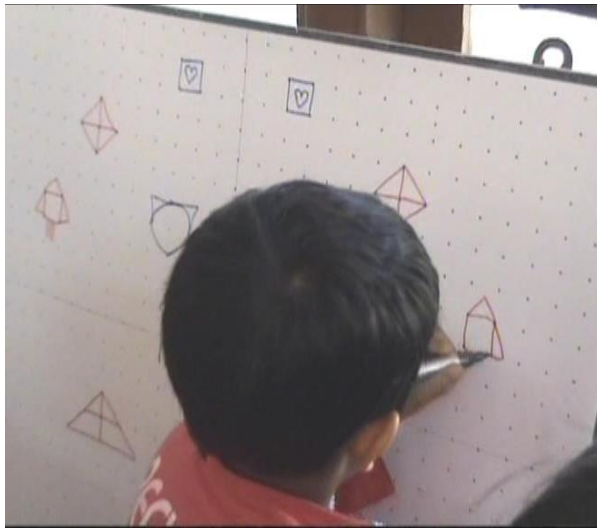
The details could be seen in the video of the workshop.

I would like to put forth some interesting observations and analysis of a few activities.

Activity no. 5: Symmetry by ink blotting-

Ink blotting was more fluid and had a surprise element thus had more similarity to play. The children could see patterns from their world of fantasy. Relating the existing thing to fantasy seemed very exciting. Trying to fit the non real things in the realm of things known. Naming the unusual patterns was fun. Verbalizing their ideas was easy as not much right- wrong was in the situation.



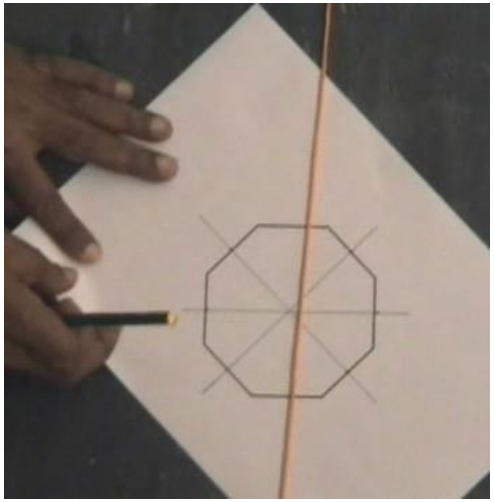


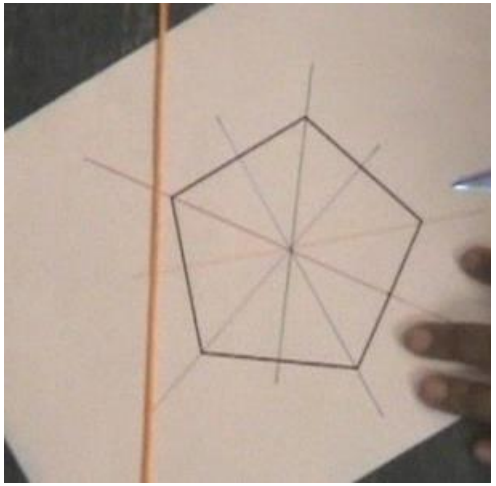
Activity no. 6: Symmetry on a grid paper –

The grid sheet was exciting as all could participate in the activity; again there was not much assessment or comparison with others. Though it would have been even more exciting if each step was interlinked with the other i.e. the object drawn by one student was taken further by the next to make another symmetric pattern. But nevertheless, the suspense prevailed till every student drew his pattern on one side and it was transformed into excitement as he tried completing the symmetry on the other side.

Activity no. 7: Symmetry through action-

Symmetry by action was not very exciting and had less participation as the symmetry was not perceived while moving. An exercise where one person stands first and then another has to stand symmetric to him would have been exciting. Or asking one person to make a symmetric composition by positioning some of his/her friends accordingly.





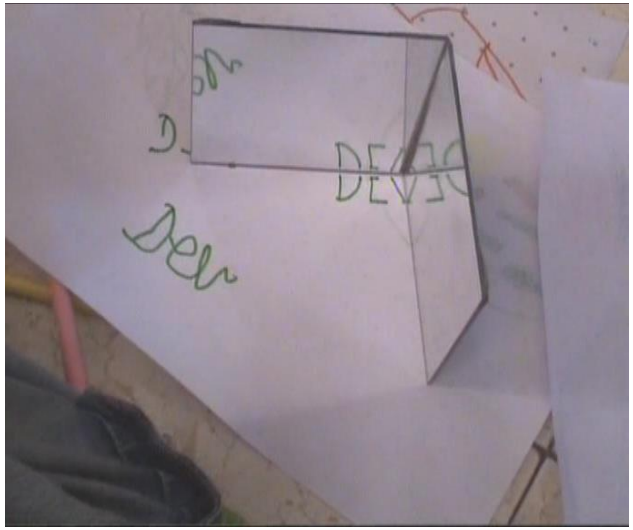
Activity no. 10: Line of symmetry-

Line of symmetry was a very exciting session for children, though full of confusion at times. Though a question should have been posed to children asking them “till now u were drawing symmetry along a line if you were given a figure and were asked to draw that line, how would u do it?” Images of faces and examples from nature for symmetry should be given to them which are more complex should be given to them. Man-made things like table, chair and other symmetric objects which are more geometric should be given, them to find their line of symmetry. This will help them understand that the line of symmetry is NOT an element of the image and is obviously not a physical line but an imaginary one. Then they should be introduced to geometrical figures. The straight vertical line put up is not a very good idea to make the children understand line of symmetry as it could create an impression that line of symmetry has to be always drawn vertical.

Using tracing paper would have been an easier option to explain the line of symmetry. As the children can see the overlap easily. They can be asked to find all possible overlaps and then discover how many times they folded to achieve the symmetry. Similarly cutouts of the polygons could have made a much easier option for children to understand the relation ship between number of sides and number of line of symmetries that occur in the figure. The attempts of children to derive a relation between number of sides/corners and number of lines of symmetry was an important step towards mathematics learning. That attempt should have been discussed in detail and a result should have been derived from it.

Activity no. 12 : mirror manipulation- grid paper

Grid paper and mirror was very engrossing for children, as there was a huge element of surprise and discovery in the exercise. The quiet self-play gave rise to many discoveries like if the angle of mirror is reduced a straight line can become different polygons. These observations could have been discussed later in a mathematics class to get the children derive why it happens so, but this platform focused more on experiential learning thus did not delve more into these discussions. Another interesting observation during this exercise was that children, specifically the girls, were spotted looking at their faces in the given mirrors. An exercise could be devised keeping ones face as a prop for self play.



Activity no. 13 : mirror manipulation- words

The exercise to write names and see whether they can be read written in half by using the mirror was also exciting. Some children really used the given mirror well to create their names. Some were a bit stuck as their names had most letters without symmetry. This session was exciting but could have been made much more vibrant if more explorations were asked for, like creating names that are symmetrical. This could give rise to a very high energy session as a lot of imagination could pour in from the children, as they could create names as existing for in the animation series they watch like shin-shan and pokemon and what not!! Similarly an exercise of finding words which are symmetrical could be introduced. Thus words that are symmetrical about vertical access would be written vertically and the ones symmetrical about horizontal access would be written horizontally. The letter X should be discussed as a special case, to assess how much children have understood about line of symmetry in the earlier exercise.

4.3.3 Analysis

The various exercises carried out emphasis the fact that children are very receptive to understanding concepts when they see it as play than study. Active involvement of children needs to be achieved by some means or other to ensure that they are learning the required concepts well.

The workshop was a tremendous success with a huge participation of the children. Similar exercises could be carried out in the classrooms. An important factor here was the absence of any assessment which is mandatory in the classroom scenario.

The exercises in a classroom scenario, or otherwise, need to be planned in such a manner that they would help children verbalize what they have discovered and make a generalization from it. They should also be probed to check if that generalization really is true and if it works in all conditions. They should be encouraged to find out the links between various observations they make.

4.4 CONCLUSIONS

Children need to feel the intrinsic need to do a certain activity in order to understand it well, thus the inclusion of play element in it becomes important.

The play kits should not be too formal. They should be flexible so that they can be used for something more than the particular exercise they are devised for.

5. FURTHER WORK

Further work needs to be done on formulating various exercises specific to a particular topic of mathematics. And different kits that can help become play objects but also teach the principles involved. Verbalization or articulation of the concepts understood is also important and should be given equal emphasis.

How children learn mathematics
by Pamela Liebeck

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