

An activity to

# Learn Exponents and Powers

for middle school students

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Interaction Design, M.Des (2015-2017)

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Guide

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# Approval sheet

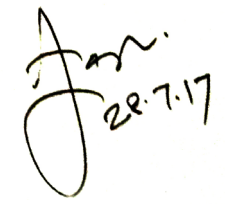
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The Interaction Design Project III entitled “An activity to learn exponents and powers for middle school students” by Herold Pazhunnana Charles, Roll Number - 156330008 is approved, in partial fulfillment of the Master in Design Degree in Interaction Design at IDC School of Design, Indian Institute of Technology, Bombay.

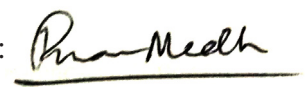
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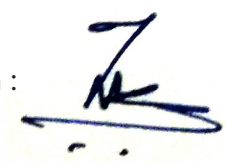
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## Declaration

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I declare that this written document represents my ideas in my own words and where others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea data/fact/source in my submission. I understand that any violation of the above will be cause for disciplinary action by the Institute and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.

A handwritten signature in black ink, appearing to read 'Herold Pazhunnana Charles', with a large, stylized flourish extending from the end of the signature.

**Herold Pazhunnana Charles**  
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July 2017



## Acknowledgments

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I would like to thank Prof. Anirudha Joshi for guiding my project by showing multiple directions to make progress and by giving continuous comments and feedback through out the project, which helped me greatly and brought different perspectives in me for thinking through a problem.

I would like to thank all the professors of interaction design Prof. Ravi Poovaiah, Prof. Venkatesh R, Prof. Girish Dalvi and Prof. Jayesh Pillai and all the other professors of IDC whose courses have helped me greatly in doing this project. I also thank all the staff of IDC for their continuous support.

I would like to thank the schools Parag Vidyalaya-Bhandup, Daffodils high school-Bhandup and KV-Powai for helping me with the primary research and evaluation of the product by allowing me to interact with their students and teachers.

I would like to thank the countless Youtubers and Online bloggers for helping me understand the technology of augmented reality and make the working prototype for this project.

Finally I would like to thank all my interaction design batch mates and my close friends in hostel for their feedback, emotional support and the motivations, without which this project could have taken years to complete.



**Herold Pazhunnana Charles**

July 2017



## Abstract

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In the middle school years, the period when mathematics concepts become harder, the drop out rates of students increases in India. The goal of the project is to develop ways in which students could learn abstract concepts in mathematics, in a concrete interactive manner with the real world. To realize this a middle school topic of “Exponents and Powers” was chosen for study.

A study on the content of the topic and current teaching methods revealed interesting user needs in the area of visualizing, remembering and using the concepts and a need to have a better connect with the real world. The solution of this project is an in-home activity using augmented reality enabled smart-phone application along with common physical objects like erasers, ice-cream sticks etc. The application has the capability to augment duplicates of the objects detected in the real world with the smart-phone camera and use those visualizations to perform various tasks with respect to exponents and powers. These visualization which are augmented with the real world objects bring a better connect to learn abstract concepts by enriching memory recall and improving emotional connect with the subject. The application has a small video window, where the instructor walks you through the content and associated physical tasks. The solution was evaluated for its effectiveness to make the students gain cognitive abilities as follows : Remember-fact, Remember-concept, Use-concept, Remember-procedure and Use-procedure. The evaluation results ascertaining the improvements in memory recall and emotional connect with mathematics, were used to conclude the effectiveness of this solution.



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# 1. Introduction

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Mathematics is a subject with vast application in the real world ranging from buying mangoes in a street shop to launching a rocket to mars. But these applications can be only realized when one understands the concepts and procedures of mathematics which are not always very real and visual. The concepts are mostly abstract in nature and the levels of abstraction and complexity increases during the middle school period starting from 6th - 7th standard onwards. Due to this many students have had difficulty in understanding them and dread this subject. Hence in an effort to understand the problems where students find issues with learning mathematics, a topic in 6-7th standard syllabus namely “Exponents and Powers” was picked.

Exponents and powers was picked because, these are the early concepts in mathematics which are explained through abstract representations and variables, which is key to define the laws of exponents. Hence students need to get clarity in these early concepts to build upon the much complex concepts in higher standards.

## Motivation

Personally while learning in school, I never used to have fun with mathematics, but used to enjoy learning mathematics concept from other subjects like science and apply them for science exhibitions in school. I had many friends who would fail miserably in math as they had difficulty in even understanding and visualizing the questions during exams. This develops fear for this subject and continues during higher education, where applications of mathematics is vast. Hence to remove this fear of mathematics from where it starts, mainly during the middle school period and enjoy

mathematics in its different form, be it music notes, art or history motivated me to start this project.

## Process

The project started with the content analysis of the topic “Exponents and Powers” using M. David Merrill’s [1] component display theory classified as the Performance-content matrix. Acquiring the structure of the content to be taught, primary research was done by interacting with middle school students and teachers, on mathematics learning difficulties.

After gathering insights from them, secondary research was done on the various existing methods of teaching like textbook resources, interactive web content, video tutors like Khan academy, BYJU’s etc to access the differentiating factors and need gaps. This was followed by understanding the different learning theories like - Behaviorism, Cognitivism, Constructivism, Constructionism, Bloom’s taxonomy etc. to derive relevant design implications for the project.

After restating the goal of the project and gathering design implications on the problems from study, design ideas were generated. Assessing the problems being solved by these ideas, the final idea was chosen and is an in-home activity with augmented reality application on smart-phone along with tangible objects. The evaluation of the activity is done individually on each student in home/class with the guidance of parents/teacher with a pretest and post-test evaluation to measure the effectiveness of the solution.

## 2. Content analysis

The first area to explore was the content and structure of the topic “Exponents and Powers”. To do this analysis, M David Merrill’s component display theory was used to classifying the content with respect to the performance-content matrix. [1] According to this theory the content is classified into four types namely -

**1. Facts** - Arbitrarily associated pieces of information such as a proper name, a date, an event, the name of a place, or the symbol used to name a particular objects, parts or events. e.g. What is the value of G, the gravity constant?

**2. Concepts** - Group of objects, events or symbols that all share some common characteristics and that are identified by the same name. e.g. What characteristics distinguish Impressionist painting from Renaissance painting?

**3. Procedures** - Ordered sequence of steps necessary for the learner to accomplish some goal, solve a particular class of problem, or produce some product. e.g. What are the steps required to find current in the DC circuit?

**4. Principles** - They are explanations or predictions of why things happen in the world. Principles are those cause-and-effect or correlation relationships that are used to interpret events or processes. e.g. As concisely as possible, explain the first law of motion.

The following is the performance-content matrix from the descriptive component display theory to assess an instruction.

				Find
				Use
				Remember
Fact	Concept	procedure	principle	

Fig 1. Performance-Content matrix

**1. Remember** - It is that performance requiring the student to search for memory in order to reproduce or recognize some item of information previously known. e.g. What is the value of ( $\pi$ )?

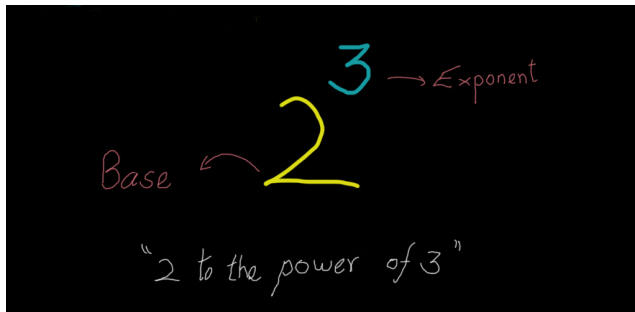
**2. Use** - It is that performance that requires the student to apply some abstraction to a specific case. e.g. Read the following story and identify that paragraph which best portrays the story's climax.

**3. Find** - It is that performance that requires the student to derive or invent a new abstraction. e.g. Set up an experiment to assess the effect of tobacco smoke on plant growth and report your findings.

The content of “exponents and powers” was classified with respect to FACTS, PROCEDURES, CONCEPT and PRINCIPLE to identify the components to focus on in the performance-content matrix. The images below are from the KHAN ACADEMY [11] videos.

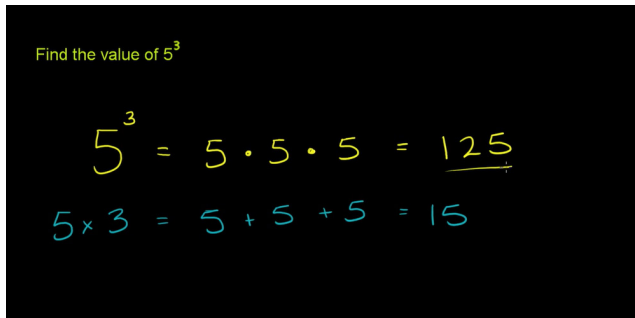
- Starting from multiplication of two numbers eg.  $2 \times 3$  and telling the difference between multiplication and exponents ie  $2^3$ .

Content Classification



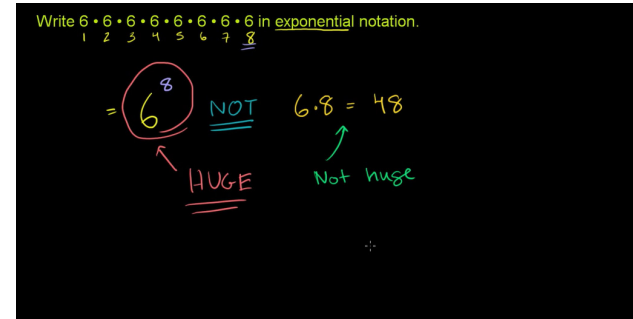
Facts

- The concept put in to question format and then answered.
- Possible misconception of exponents again clarified with wrong example and stressing the difference between  $5 \times 3$  and  $5^3$ .



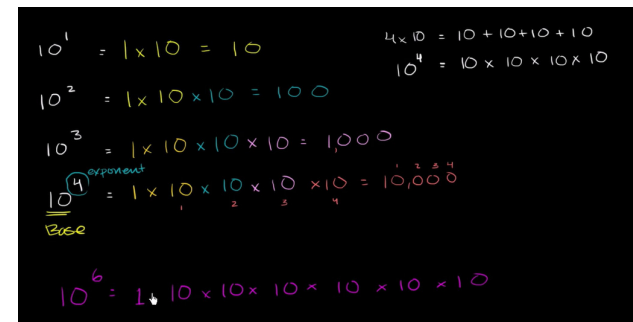
Procedure

- First expansion of exponents into numbers taught and now the opposite i.e. writing in exponents from expanded form.
- Again multiplication and exponent difference is repeated.



Procedure

- Exponents explained with base 10.
- Again the possible mistake repeated.



Procedure

- Laws of exponent : multiplication

$$5^7 = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$$

$$3x \cdot 3x \cdot 3x = (3 \cdot 3 \cdot 3)(x \cdot x \cdot x)$$

$$(3x)^3 = 3^3 x^3 = 27x^3$$

$$6^3 \cdot 6^6 = \underbrace{6 \cdot 6 \cdot 6}_3 \cdot \underbrace{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6}_6 = 6^{3+6} = 6^9$$

$$2^2 \cdot 2^4 \cdot 2^6 = 2^{(2+4+6)} = 2^{12}$$

Procedure

- Laws of exponent : division

$$5^6 / 5^2 = 5^{6-2} = 5^4$$

$$\frac{6^7}{6^3} = 6^{7-3} = 6^4$$

$$\frac{3^4}{3^{10}} = \frac{\cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3}}{\cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3}} = \frac{1}{3^6}$$

$$\frac{a^b}{a^c} = a^{b-c}$$

$$\frac{3^4}{3^{10}} = 3^{4-10} = 3^{-6} = \frac{1}{3^6}$$

$$\frac{a^b}{a^c} = \frac{1}{a^c}$$

Procedure

- Concept of replacing numbers with abstract x,y or a,b etc introduced.

$$6^3 \cdot 6^6 = \underbrace{6 \cdot 6 \cdot 6}_3 \cdot \underbrace{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6}_6 = 6^9$$

$$2^2 \cdot 2^4 \cdot 2^6 = 2^{(2+4+6)} = 2^{12}$$

$$x^2 \cdot x^4 = x^6$$

$$x \cdot x \cdot x \cdot x \cdot x \cdot x$$

$$(a^3)^4 = a^{3 \cdot 4} = a^{12}$$

$$a^3 \cdot a^3 \cdot a^3 \cdot a^3 = a^{3+3+3+3} = a^{3 \cdot 4} = a^{12}$$

Concept

- Laws of exponent : Power of a power.

$$(ab)^4 = (ab)(ab)(ab)(ab) = \underbrace{a \cdot a \cdot a \cdot a}_4 \cdot \underbrace{b \cdot b \cdot b \cdot b}_4$$

$$(ab)^c = a^c b^c$$

$$(a^3)^2 = a^3 \cdot a^3 = a^{3+3} = a^6$$

$$(a^b)^c = \underbrace{a^b \cdot a^b \cdot a^b \cdots a^b}_c = a^{b \cdot c}$$

Procedure

Exponents with zero power and negative bases explained.

$(-2)^3 = (-2) \cdot (-2) \cdot (-2) = -8$   
 $= 1 \cdot (-2) \cdot (-2) \cdot (-2) = -8$

$2^0 = 1$   
 $2^1 = 1 \cdot 2 = 2$   
 $2^2 = 1 \cdot 2 \cdot 2 = 4$   
 $2^3 = 1 \cdot 2 \cdot 2 \cdot 2 = 8$

$(-3)^3 = (-3) \times (-3) \times (-3) = -27$

(neg)<sup>even</sup>  $\Rightarrow$  pos  
 (neg)<sup>odd</sup>  $\Rightarrow$  neg

$-4^2$	$(-4)^2$	$-2^3$	$(-2)^3$
$-(4 \times 4)$	16	-8	-8
-16			

- Introduction of decimals and steps to convert them to exponents or scientific notation.

$7.4 \times 10^4$   
 $74,000 = 74,000$   
 $120,000 = 1.2 \times 10^5$   
 $1.2 \times 100,000$   
 $12. = 1.2 \times 10^1$

$0.00175$   
 $0.00175$   
 $1,765,244.$   
 $1.765244 \times 10^6$   
 $0.00281 \times 10^3 \times 10^{-3}$   
 $2.81 \times 10^{-3}$

Procedure

- Intuition of negative exponents, on how mathematicians would have decided this rule.

$a^{-b} = \frac{1}{a^b}$

The diagram shows a sequence of powers of  $a$  from  $a^{-2}$  to  $a^4$ . Arrows indicate the relationship between adjacent terms:  $a^{-2} \times a^{-1} = a^{-1}$ ,  $a^{-1} \times a^0 = a^0$ ,  $a^0 \times a^1 = a^1$ ,  $a^1 \times a^2 = a^2$ ,  $a^2 \times a^3 = a^3$ , and  $a^3 \times a^4 = a^4$ . A box at the bottom contains the formula  $a^{-b} = \frac{1}{a^b}$ .

Concept

From the classification it is evident that the content has a lot of facts, procedures and two concepts to be learned. The objective of the classification is to identify the facts, procedures and concepts that needs to be remembered and used by the students in different scenarios or applications.

Procedure

### 3. Primary Research

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As part of the primary research, students and teachers of middle school were interviewed to gather insights on the problems in learning mathematics. Total of two teachers and five students were asked contextual questions on learning mathematics generally and specifically with respect to the topic of exponents and powers.

#### Insights :

The students had a general aversion for mathematics and feared most during an examination as understanding the math questions itself was a big struggle. Hence many students go to mathematics tuition after the school hours.

The teachers also find it difficult to make the students learn the mathematics concepts and mostly adopt the way of repeated practice to help the students retain the procedures in memory. This practice in turn make the students mug up the procedures without understanding the concept. They tend to solve questions similar to the example problems, but when the question is twisted a bit, they struggle to apply the same concepts. With respect to the topic of exponents and powers, teachers had pointed out that students generally confuse exponents with multiplication/division and sometimes get stuck while applying the laws of exponents on a tougher question.

In order to relate the math problems with practical scenarios, teachers have employed different methods like physical activities and group quiz etc. But they were not sure with this approach of teaching out of the textbook ways. This was because usually the students show initial excitement and enjoy the task, but when it comes to exams the teachers doubt whether this effect gets converted into marks or not.

Hence from the primary research it was indicative that there was a lack of varied approaches available to the teachers to make the students understand the mathematics concepts, apart from the standard methods of repeated practice.

## 4. Secondary Research

### 4.1 Existing methods of teaching

The content of various teaching resources of the topic exponents and powers were analyzed and the key differentiating factors and need gaps were noted. Following are the sources of the content which were analyzed -

1. NCERT/CBSE syllabus
2. Tamilnadu state board syllabus
3. Madhya Pradesh state syllabus
4. Khan academy
5. BYJU's application
6. Mathisfun website
7. Wisdom leap website
8. Learning math through history
9. Power of Ten by Charles and Ray Eames
10. Smartlearn forall youtube channel

The following section details the key differentiating elements in the learning/teaching style and concludes with a comparison between the sources to gather insights.

#### 1. NCERT/CBSE syllabus : [8]

- Diagrams were used extensively to grab the attention of students.
- Examples with caution box for students to avoid errors of misconceptions.

**Problem** - Less input about the concept intuition in the course content.

#### 13.1 INTRODUCTION

Do you know what the mass of earth is? It is 5,970,000,000,000,000,000,000,000 kg!

Can you read this number?

Mass of Uranus is 86,800,000,000,000,000,000,000,000 kg.

Which has greater mass, Earth or Uranus?

Distance between Sun and Saturn is 1,433,500,000,000 m and distance between Saturn and Uranus is 1,439,000,000,000 m. Can you read these numbers? Which distance is less?

These very large numbers are difficult to read, understand and compare. To make these numbers easy to read, understand and compare, we use exponents. In this Chapter, we shall learn about exponents and also learn how to use them.



#### 13.2 EXPONENTS

We can write large numbers in a shorter form using exponents.

Observe  $10,000 = 10 \times 10 \times 10 \times 10 = 10^4$

The short notation  $10^4$  stands for the product  $10 \times 10 \times 10 \times 10$ . Here '10' is called the **base** and '4' the **exponent**. The number  $10^4$  is read as **10 raised to the power of 4** or simply as **fourth power of 10**.  $10^4$  is called the **exponential form** of 10,000.

We can similarly express 1,000 as a power of 10. Since 1,000 is 10 multiplied by itself three times,

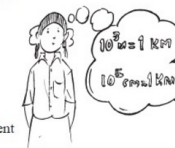
$$1000 = 10 \times 10 \times 10 = 10^3$$

Here again,  $10^3$  is the exponential form of 1,000.

Similarly,  $1,00,000 = 10 \times 10 \times 10 \times 10 \times 10 = 10^5$

$10^5$  is the exponential form of 1,00,000

In both these examples, the base is 10; in case of  $10^3$ , the exponent is 3 and in case of  $10^5$  the exponent is 5.



#### Caution!

Consider  $2^3 \times 3^2$

Can you add the exponents? No! Do you see 'why'? The base of  $2^3$  is 2 and base of  $3^2$  is 3. The bases are not same.

## 2. Tamilnadu state board syllabus : [9]

- Explanation of the topic squares and cubes in exponents and powers start with a story from history of a great mathematician from India - Srinivas Ramanujan. This was a unique way to retain the concept in memory with the help of a story.

### 1.7.3 கனங்கள்

#### அறிமுகம்

புகழ்பெற்ற கணிதமேதை இராமானுஜன் அவர்களின் வாழ்வில் நடைபெற்ற ஒரு முக்கிய நிகழ்வைப் பற்றிக் காணலாம்.

ஒரு முறை கணித வல்லுநர் பேராசிரியர் G.H. ஹார்டி அவர்கள் திரு. இராமானுஜன் அவர்களைப் பார்க்க வாடகை மகிழ்வந்தில் வந்தார். அவர் வந்த வாடகை மகிழ்வந்தின் எண் 1729. இருவரும் பேசிக் கொள்ளும்போது ஹார்டி அவர்கள் தான் வந்த வாடகை மகிழ்வந்தின் எண் 1729 என்றும், அது ஒரு “மந்தமான எண்” என்றும் கூறினார். உடனே இராமானுஜன் அவர்கள் 1729 என்பது மிகவும் அற்புதமான எண் என்றும், அவ்வெண்ணானது இரு கன எண்களின் கூடுதலாக இரு வெவ்வேறு முறைகளில் எழுதக்கூடிய மிகச்சிறிய எண் எனவும் விளக்கினார்.

$$\text{அதாவது, } 1729 = 1728 + 1 = 12^3 + 1^3$$

$$\text{மற்றும் } 1729 = 1000 + 729 = 10^3 + 9^3$$

1729 ஐ இராமானுஜன் எண் என்று அழைக்கிறோம்.

இப்பிரிவில் கனங்கள், கன மூலங்கள் மற்றும் அதனுடன் தொடர்புடைய உண்மைகளைப் பற்றிப் பார்ப்போம்.

#### கன சதுரம்

நாம் வடிவியலில் கனம் என்ற வார்த்தையைப் பற்றிப் படித்துள்ளோம். நீளம், அகலம், உயரம் ஆகிய அனைத்தும் சமமாக உள்ள ஓர் கன உருவம் கன சதுரம் ஆகும். ஒரு கன சதுரத்தின் ஒவ்வொரு பக்கமும் 'a' அலகுகள் எனில் அதன் கன அளவு  $a \times a \times a = a^3$  கன அலகுகள்.

$a^3$  என்பதை a இன் “மூப்படி” அல்லது “a இன் கனம்” என அழைக்கலாம்.

இப்பொழுது, 1, 8, 27, 64, 125, ... என்ற எண்களைக் கவனிக்கவும்.

இவை “கன எண்கள்” அல்லது “முழு கன எண்கள்” என அழைக்கப்படுகின்றன.



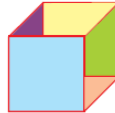
சீனிவாச இராமானுஜன் (1887-1920)

ஈரோட்டில் பிறந்த இந்தியக் கணித மேதையான இராமானுஜத்தின் “எண்ணியல் கோட்பாடுகள்” குறித்த அவரது பங்களிப்பு அவருக்கு மிகப்பெரும் உலகப் புகழைப் பெற்றுத் தந்தது. மிகக் குறுகிய அவரது வாழ்நாட்களுக்குள்ளேயே சுமார் 3900 ஆராய்ச்சி முடிவுகளைத் தனிபாகவே தொகுத்து வெளியிட்டுச் சாதனை படைத்துள்ளார்.



#### னீர் அறிவீர்?

1729 என்ற எண்ணானது மிகச் சிறிய இராமானுஜன் எண்ணாகும். இதேபோன்ற வேறு சில எண்கள் 4104 (2, 16 : 9, 15), 13832 (18, 20 : 2, 24).



## 3. Madhya Pradesh state board syllabus : [10]

- Problem** - The syllabus heavily lacked the use of adequate examples that apply in the real world and does not handle the misconceptions and error that are possible while learning the topic. For example the exponents with different bases cannot be multiplied using the laws of exponents for multiplication, which are the errors that are usually made by students.

पाठ 4

घातांक

#### आइए सीखें-

- घातांक, आधार की परिभाषा।
- घनात्मक पूर्णांक घातवाली परिमेय संख्याओं का विस्तारित रूप।
- घातांक के नियम एवं शून्य घातांक।
- ऋणात्मक पूर्णांक घातांक के रूप में।
- बड़ी एवं छोटी संख्याओं को घातांक का उपयोग करते हुए लिखना।
- संख्याओं का वैज्ञानिक संकेतन।

हम संख्याओं के गुण से परिचित हो चुके हैं। यदि एक ही संख्या को उसी संख्या से बार-बार गुणा किया जाये तो इसे अलग प्रकार से भी व्यक्त किया जा सकता है। इस पाठ में हम घातांक एवं उससे संबंधित नियमों का अध्ययन करेंगे।

#### 4.1 घातांक

हम जानते हैं कि  $10 \times 10 \times 10 = 1000$

यहाँ 10 का 10 से 3 बार गुणा किया गया है।

इसे  $10^3$  के रूप में भी लिखते हैं। इस प्रकार

$$10^3 = 1000$$

इसी प्रकार  $2^4 = 2 \times 2 \times 2 \times 2$

$$(-5)^3 = (-5) \times (-5) \times (-5)$$

$$p^4 = p \times p \times p \times p$$

$10^3$  को हम 10 की घात 3 कहते हैं।

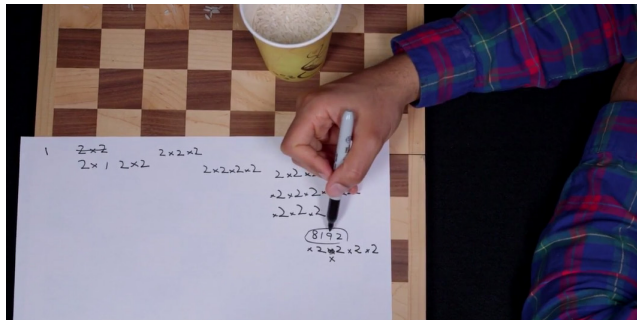
इसी प्रकार

$$p \times p \times p \times p \times p = p^5$$

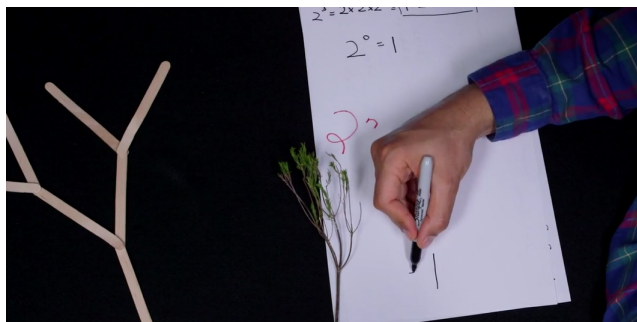
$p^5$  में p आधार (base) तथा 5 को घातांक (index) कहते हैं। इसी प्रकार  $a^m$  में a आधार तथा m घातांक है।

4. Khan academy : [11]

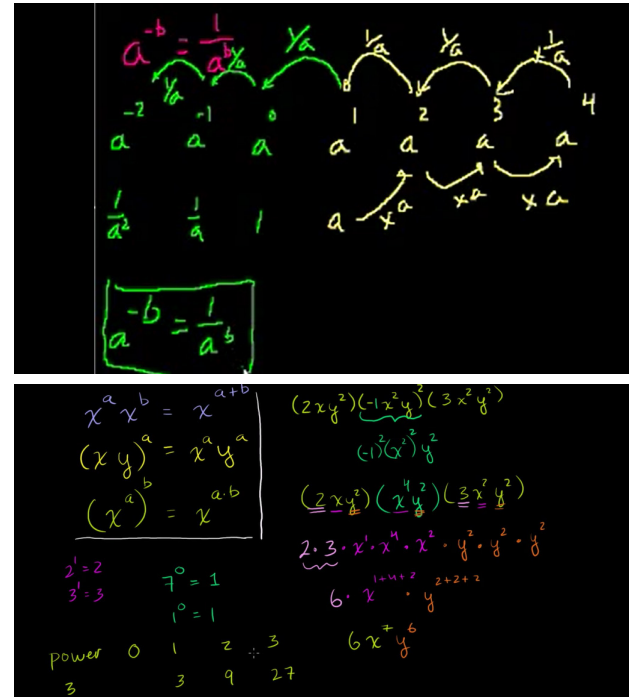
- Intro with examples like rice grouping on chess board, with number of rice being doubled from one box to another till the 64 boxes in chess board. Interesting example application acts as a use-concept component of the performance-content matrix.



- Another real world comparison with objects, is with ice cream stick used to make tree structure and explain exponents using child branches. Repetition is incorporated for examples as well as generic errors that students might commit.



- Intuition of negative exponents, on how mathematicians would have decided this rule, is explained well as a concept.



- The use of live audio to explain the long procedures step by step on a black background which resembles like writing in a physical black board is an effective way of communication.

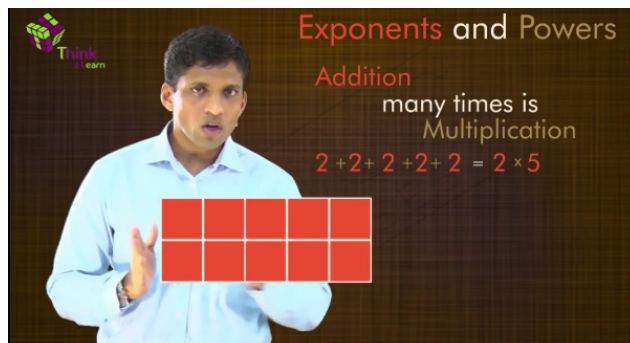
**Problem** - Lack of visualizations while explaining the procedures which could aid memory recall.

## 5. BYJU's application : [12]

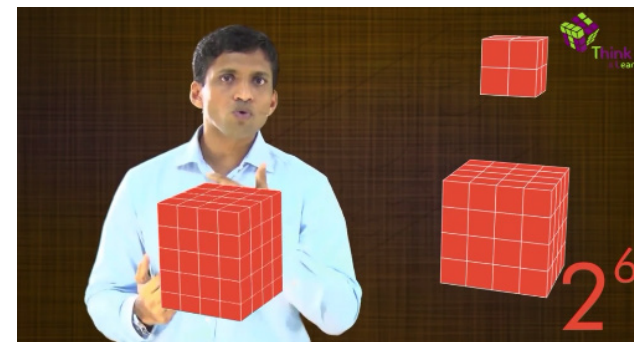
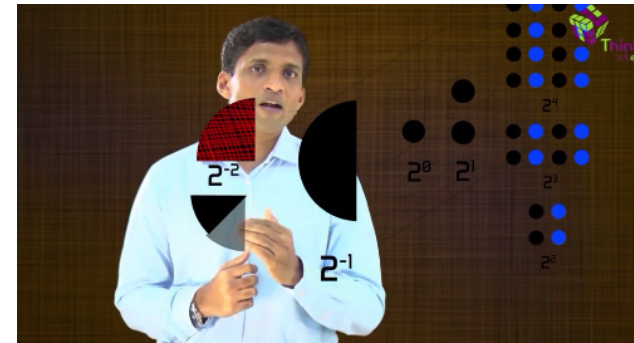
- The key teaching style is to maintain the eye contact of the teacher with the viewer and the content in progress be displayed in front of them through visual graphics.
- Hand and eye movements coordinated with the graphics to give a realistic and responsive content while teaching.



- Instead of just numbers written like in Khan academy, visual shapes and graphics like rectangles, cubes etc. are floated on screen for students to get a more visual understanding of the concept.



- Visualization of exponents in 2D shapes and 3D cubes act as a strong method to understand the concept in use.



**Problem** - Even though the graphics of 3D cubes were effective in getting a mental picture of the concept, the examples did not relate to real world objects so as to relate with an emotion and act as a better memory recaller.



### 9. Charles and Ray Eames - Power of Ten : [16]

Learning concept of exponents and powers through moving visuals which are relatable with the real world in comparable scale and proportions.



### 10. Smart Learning for All youtube channel : [17]

Story started and then in middle of the scene, the topics of exponents are explained. The story act as a memory recaller.



### Content comparison across textbook sources :

The content is compared with the degree of facts, concepts and procedures dealt in each board along with the sequence of teaching the topics.

Content	NCERT/CBSE	TN state board	MP state board	Content type
Exponents & powers notations				Fact
Application examples	1		9	Concept
Question n Answer format	5	3	3	Fact
Clear misconception	6			Concept
Expansion of exponents	4	1	4	Procedure
error repeated				Procedure
Exponents with Base 10	2	4	1	Procedure
laws of exponents multiplication	7	5	5	Procedure
abstract variables introduced	8	2	2	Concept
harder examples	9	9		Procedure
laws of exponents Division	10	6	6	Procedure
laws of exponents parenthesis	11	7	7	Procedure
zero power and negative bases	12	8	8	Procedure
Number system revised				Fact
Decimals and scientific notations	13	10	10	Fact
Intuition of negative exponents				Concept
Squares, cubes and roots		12		Procedure
Story from history		11		Fact
Visual representation of concept	3			Concept
	Visual significance with less input about concept intuition	Square, cubes and root explained along with history connected to it	Handling the misconceptions and adequate example applications not done	

### Content comparison across online sources :

The content is compared with the degree of facts, concepts and procedures dealt in each Online content along with the sequence of teaching the topics.

Content	Khan Academy	BYJU's app	Content type
Exponents & powers notations	1	1	Fact
Application examples	2		Concept
Question n Answer format	3		Fact
Clear misconception	4	3	Concept
Expansion of exponents	5	4	Procedure
error repeated	6	5	Procedure
Exponents with Base 10	7	6	Procedure
laws of exponents multiplication	8	7	Procedure
abstract variables introduced	9	8	Concept
harder examples	10		Procedure
laws of exponents Division	11	9	Procedure
laws of exponents parenthesis	12	10	Procedure
zero power and negative bases	13	11	Procedure
Number system revised	14		Fact
Decimals and scientific notations	15	12	Fact
Intuition of negative exponents	16		Concept
Squares, cubes and roots			Procedure
Story from history			Fact
Visual representation of concept		2	Concept
	Application examples of the concept with its intuition explained	Visual representation of facts had greater significance	

The comparison across the sources reveal that the sequence of the topics are completely varied with some starting with basic notations till the laws of exponents and some starting with example applications to build up curiosity which proceed till the procedure to find negative exponents etc. Hence it is understood that there is no set sequence to deliver the content. Since it is difficult to decide the most effective sequence of delivery, first building curiosity with example facts and then relating the concepts and procedure is taken forward as the sequence to deliver the content for this project.

The key positive features of the teaching methods in textbook sources are identified as follows :

- Diagrams were used well, illustrating the key things to remember.
- Stories from history were used as an evocative content to bring more emotion to the concept.
- Possible errors were repeated in caution boxes to reinforce the correct procedure.

The key positive features of the teaching methods in online sources are identified as follows :

- 2D and 3D visualizations can help convey the abstract concept better.
- Eye contact and voice instructions online can be a good way to grab the attention of students mimicing a classroom setup.
- Interactive tools can be used to see the effect of changing the numbers live with instant result.

The most evident problems in all of the sources were the limitation it had to visualize the procedures and lack of opportunities in the content structure to help the students retain the concepts in memory, other than repeated practice of the procedures.

## 4.2 Learning theories

To understand the problems found from the existing teaching methods, various learning theories were studied. The theories looked upon were Behaviourism [2], Cognitivism [3], Constructivism [3] and Constructionism [4]. Since the most evident problems that came out were on the lack of opportunities in the content structure to help the students retain the concepts in memory, Constructivist and Constructionist theories were focused more.

### Constructivism

Founded by Jean Piaget [3], constructivism emphasizes the importance of the active involvement of learners in constructing knowledge for themselves. The opportunities for the learners to construct knowledge for themselves was limited in the existing teaching methods discussed in the previous chapter.

### Constructionism

Developed by Seymour Papert, Constructionism shares constructivism's connotation of learning as 'building knowledge structures' irrespective of the circumstances of the learning. It then adds the idea that this happens especially in a context where the learner is consciously engaged in constructing in real world, whether it's a sand castle on the beach or a theory of the universe.[4]

In the existing teaching methods even though there may be some opportunities to build knowledge structures by themselves, the missing link could be the above mentioned idea that learning happens especially in a context where the learner is consciously engaged in constructing in the real world.

The other theories and articles looked upon were as follows :

### Bloom's taxonomy explaining the various levels of learning. [5]

The following are the 6 levels of learning one can attain, which is related to specific ages or an individual's capability. For example a 2<sup>nd</sup> grade student may be expected to attain only the first level of learning i.e. remembering, whereas a PhD graduate would be expected to attain the 6<sup>th</sup> level of learning i.e. creating.

1. Remembering
2. Understanding
3. Applying
4. Analyzing
5. Evaluating
6. Creating

The chosen topic "Exponents and powers" is taught to 7th standard students and with respect to the Bloom's taxonomy this is the stage of remembering, understanding and applying the concept which reaches till stage 3. Hence at this age the students are expected to reach at-least 3<sup>rd</sup> stage of learning.

### Turkle S. Evocative objects - Things we think with. [6]

The author of this article explores the idea of evocative objects. Most objects exert their memory holding power because of the particular moment and circumstances in which they come into one's life. These objects could evoke certain memory when seen or

thought about again. For example, a violinist could be evoked of a memory with her teacher, by seeing or thinking of that specific violin which she had on her teachers birthday and had learned the fundamentals of composing music that day. Here that violin acts as an evocative object which evokes the memories of her teacher as well as the fundamentals of music theory. Hence the idea of evocative object could be used to evoke memories of the mathematics concepts learned.

### **Michael Eisenberg, Mindstuff: educational technology beyond the computer [7]**

The paper is an extension of the ideas of Seymour Papert's book "Mindstorms" where he shares his experience with gears of cars, through which he fell in love with mathematics. Here the gears acted as a "transitional object" to connect the abstract world with the physical one and acted as a cognitive bridge. But then he argues that gears may not become the transitional object for all. He then identifies the possibilities of new technologies and computers as a transitional object which could simulate a variety of objects by itself.

A transitional object can provide emotional affordance. For example rather than using a paper tape as transitional object to learn geometry, wool weaving or bead construction could effectively teach geometry better, which could also be worn. With this concept the author suggest 3 strategies to achieve this -

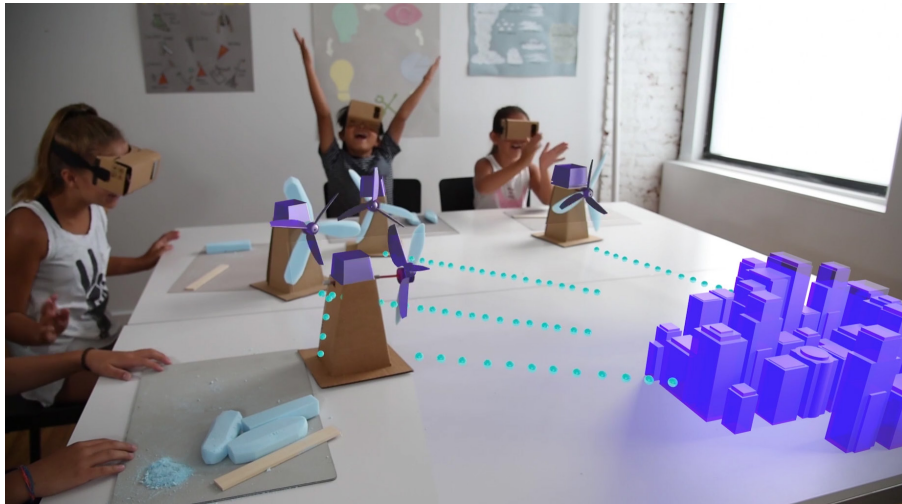
1. The development of software applications to augment or enrich children's use of tangible materials.

2. Embedding computational capabilities within physical objects. They may also be combined into larger systems, "kits", or sets of mutually communicating or inter-operable objects.
3. The exploration and use of a wide variety of new materials of various levels of "intelligence", adaptability, expressiveness, educational interest, or potential for integration with computational techniques.

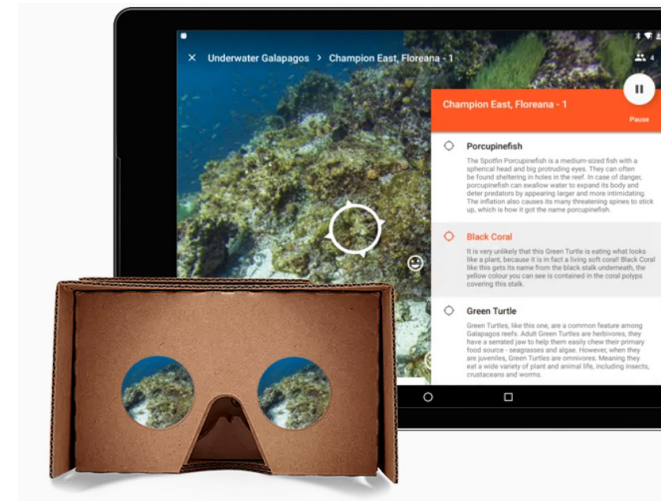
In other words, the students can engage in new sorts of craft projects with the aid of computers to learn mathematics.

## 4.3 Educational technology

wPeer AR tool - Augmented reality as a tool to visualize. [18]



Google Expeditions - A virtual reality application as educational guide. [20]

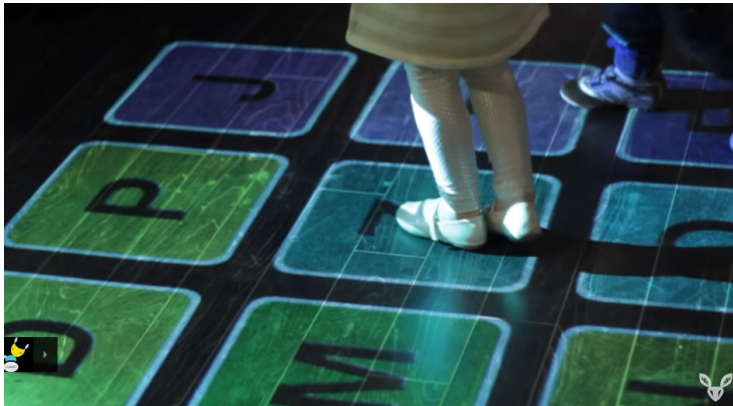


OSMO - Augmented reality app for Ipad. [19]

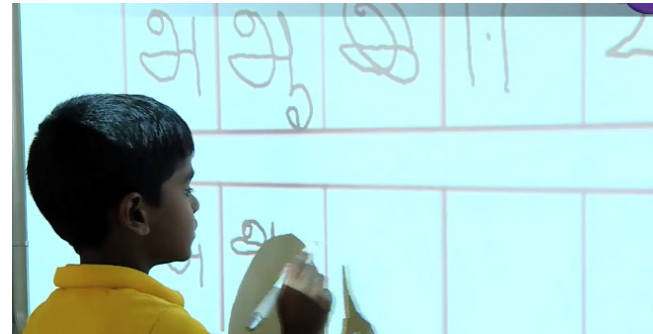


The applications like wPeer, OSMO and Google Expeditions have started exploiting the possibilities of new technologies such as the Augmented reality and Virtual reality to act as a tool for learning by combining physical activity and integrating the real world with the content to be delivered to the students.

**LUMO - Interactive Projections on the floor for learning. [22]**



**BenQ Interactive Projector - Activity oriented interactive board [21]**



**Little bits - An plug an play electronics modular kit for constructionist learning. [23]**



The other education technologies incorporated in schools today are the interactive projectors on floors or boards, to deliver media rich content and to deliver collaborative activities among the students.

Products like Little bits are also used as a physical teaching kit to do different kind of creative tasks by the students and learn the concepts while doing them.

## 5. Problems and Opportunities

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### Unexplored options

- Example activities involving physical or tactile methods are very less, which could act as a transitional object and help in memory retainment.
- Idea of substituting or generalizing numbers into variables are not visualized in any of the analyzed content.
- Interactive way of response for an action as a mode of learning and understanding not explored.
- Even though the application of exponents to diameter of earth or radius of blood cell etc. are relevant, the possibility of applying them to more relatable and day to day objects which are around them are not explored.
- Other subjects like history in case of Ramanujan example in TN state board text book are related with the topic of exponent very briefly, but not as a story where the concept is blended into it.
- Retrospection of the idea of substituting big numbers to smaller form of representation is not done. Eg representing written numbers like Eight thousand nine hundred and twenty five as 8925 or real world examples like Shoe rack token in temples or bag in shopping mall where big is represented by small token coin.

### Problems

- The lack of opportunities in the content structure to help the students understand the concepts and retain them in memory, rather than by the repeated practice of procedures.
- Limitations of the textbook medium to interactively visualize the procedures in the content structure.
- Students have a general aversion for mathematics and fear most during an examination as understanding and visualizing the math questions itself was a big struggle.
- Squares, cubes, square root and cube root not dealt along with this topic in most of the syllabus.
- More error cases not explained, like while dividing exponents, canceling the exponents instead of digits are some mistakes students make.

### Opportunities for ideation

- A physical activity for students which could act as a memory retainer and also make them understand the concept in context of the world around them.

- Design in-home activities as a tutor after school, under the guidance of parents to assist in visualizing the concept and leverage the technologies available in home.
- Students need tools to visualize or imagine exam questions to understand the problem and attempt it without fear.
- Use sound as a medium to convey the concept. For example by varying the pitch frequency or loudness.
- How to represent a variable in physical world.
- Collaborate stories which are not just the history of math but which also acts as a tool to collaborate current stories with the math concepts to evoke emotions and connect memories with the concepts.
- Design a physical or virtual game to convey the concept of exponents in visual and fun way which is relatable with the world around them.
- The stories to merge with learning mathematics can't be long as students can get sleepy if not interesting and could become distracted from the main concept if not balanced.
- The time allotted for a topic with respect to the syllabus in school is limited and hence the solution has to be in pace with existing durations.

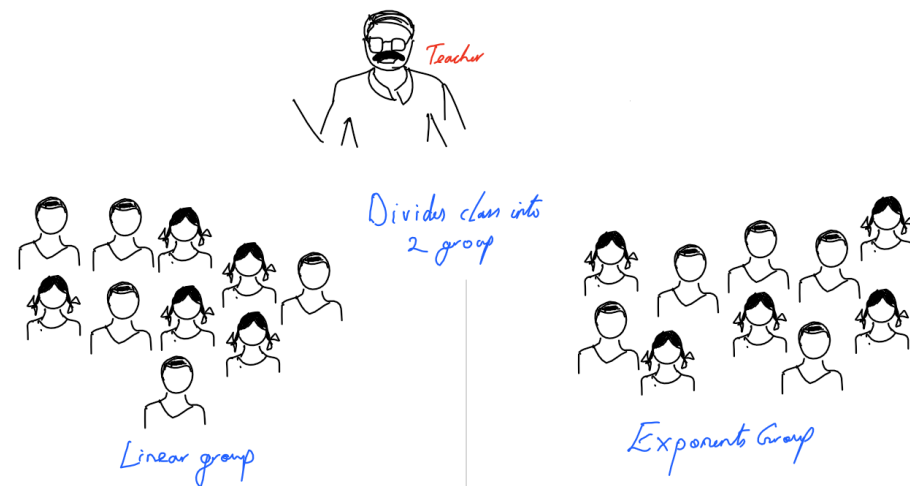
### Constraints

- The material involved in the physical activities must be cheap and available in stores easily. It also could be things around them, which students might use often.
- The technology to be used must be cheap or it must be at least accessible to the students from their parents in home.

## 6. Ideation

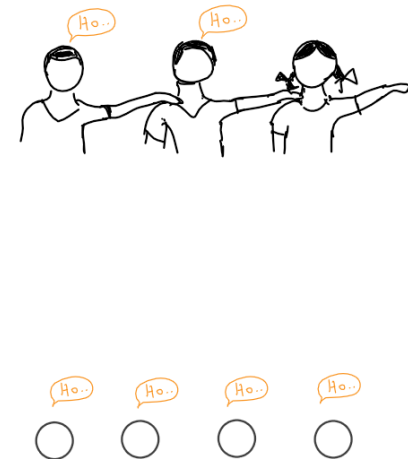
### Idea 1 : Class activity with sound

An activity to make students learn the concept of exponents through physical activity and sound.

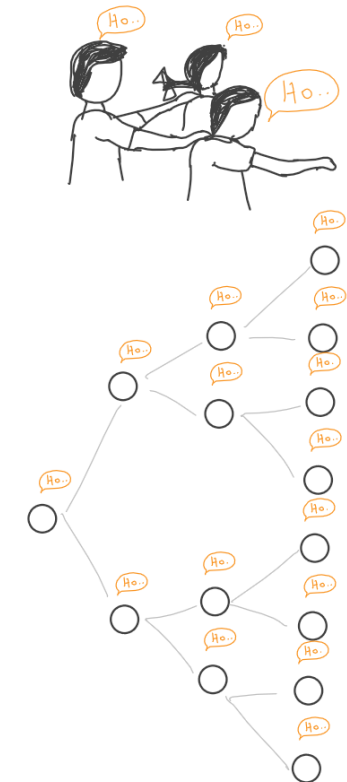


When "start" said by teacher each student should tap on only one another classmate and he/she shouts "HO".

When "start" said by teacher each student should tap on two other classmates and all should shout "HO".



In four steps of the tapping, the loudness of the sound "HO" increases, but steadily.



With in four steps of the tapping, the loudness of the sound "HO" increases tremendously.

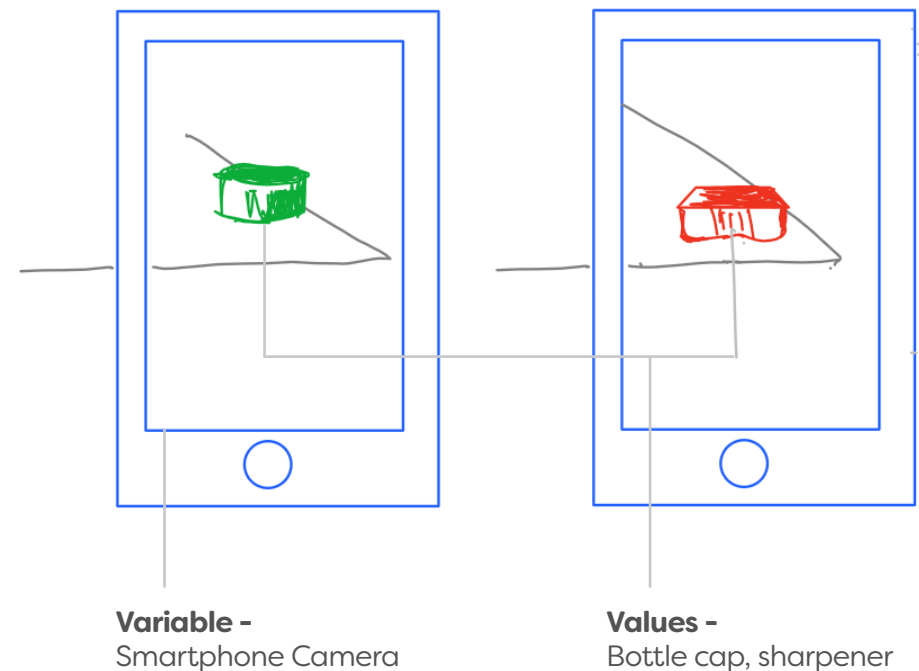
Hence through the tremendous increase in “HO” sound we can convey the concept of exponential increase in value and significance of powers in numbers representation through auditory senses and physical activity to have better retention in their memory.

### What problem and needs does this Idea address?

- The lack of opportunities in the content structure to help the students retain the concepts in memory, other than repeated practice is dealt using physical activity.
- Limitations of the textbook medium to visualize the content is tried using the medium of sound.

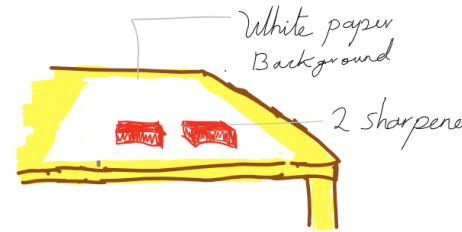
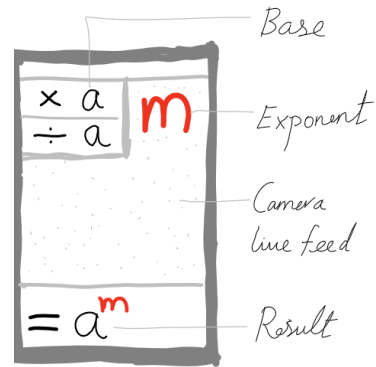
## Idea 2 : Augmented reality app

- Use augmented reality in smartphone to accommodate very large/small numbers and represent variable by connecting physical and virtual world.
- Need to visualize variable in physical world. Mobile with camera acts as a container or variable to hold different objects/variables within its screen.
- The object to be used as values for the variables could be any easily available classroom materials like sharpener or water bottle lid etc.



- Waste objects or small things that could be found inside/besides a classroom as tangibles :  
Water bottles, Crushed papers, Ruler, Geometry box, Chair, tables, Bricks & stones, Soil, Water, Ink bottle and ink filler, Balloons, Thermacool, Thread, Paper, Needle, Sticky notes.
- Using the Augmented reality technology in Mobile, an app is developed to visualize the tangibles objects in exponential form, by duplicating them in the real world.

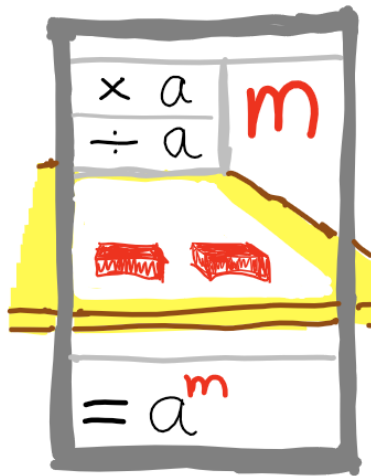
Augmented reality app -



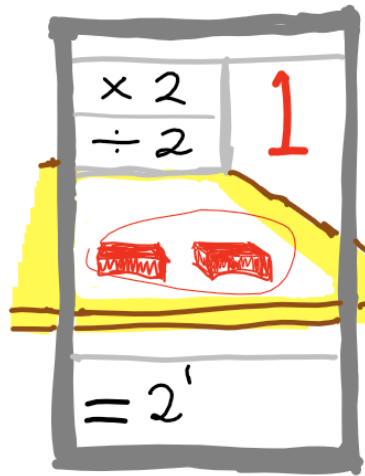
Tangible object on white paper

Object recognized, 'a' replaced by '2'

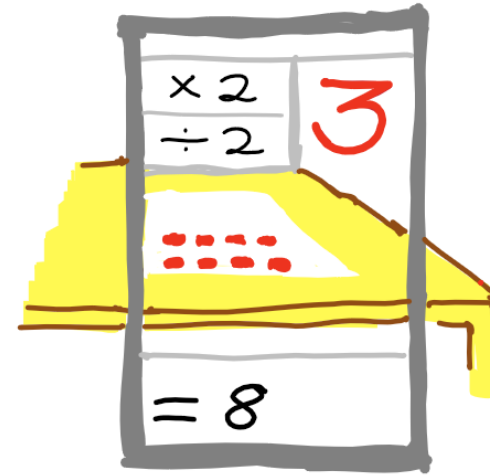
'x2' pressed twice to make exponent '3'



AR app camera brought near the object for detection.

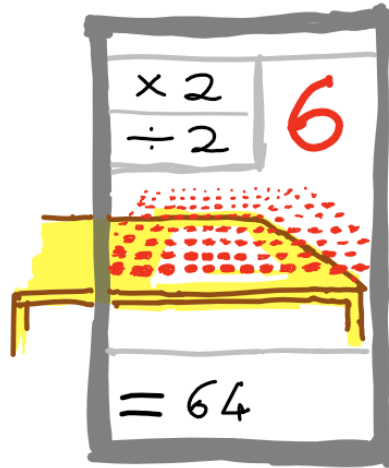


Two sharpeners in white background traced by image recognition and 'a' the base replace as '2'.

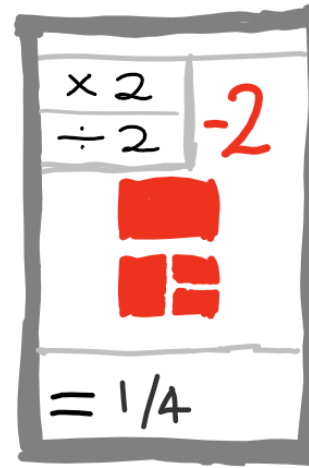


Two sharpeners shown as eight in perspective on white paper using augmented reality.

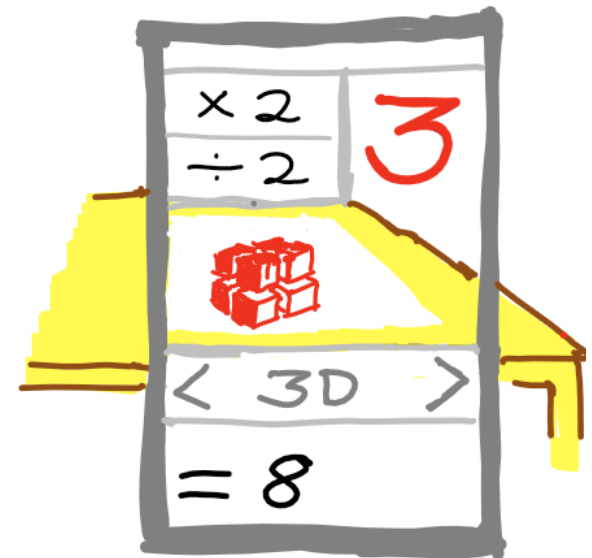
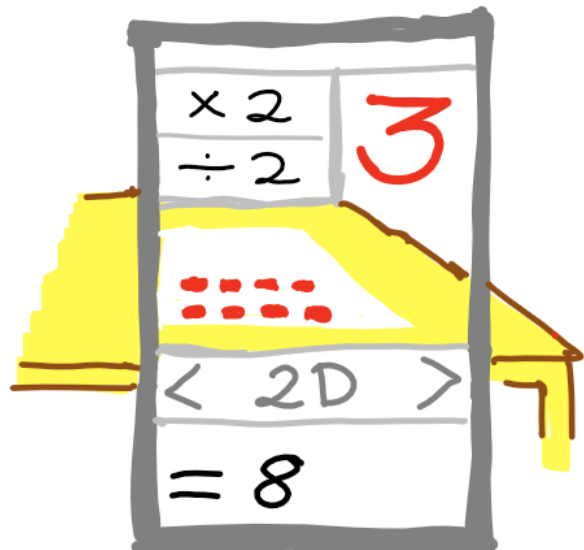
Move away from the object to visualize larger exponents



Move close to the object or hand zoom to Visualize negative exponents

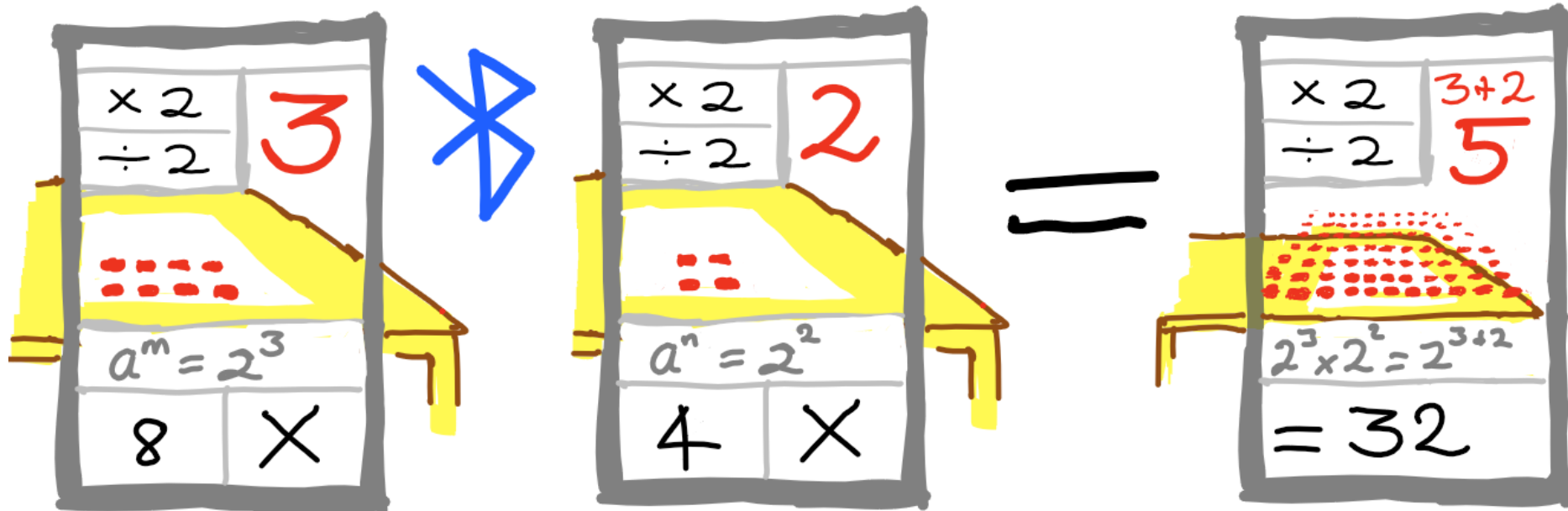


Multiple visualizations of the Augmented reality could also be chosen -



To **apply the same concept to Laws of exponents** with smartphone screen as variable, we need two smartphones which could be paired using Bluetooth.

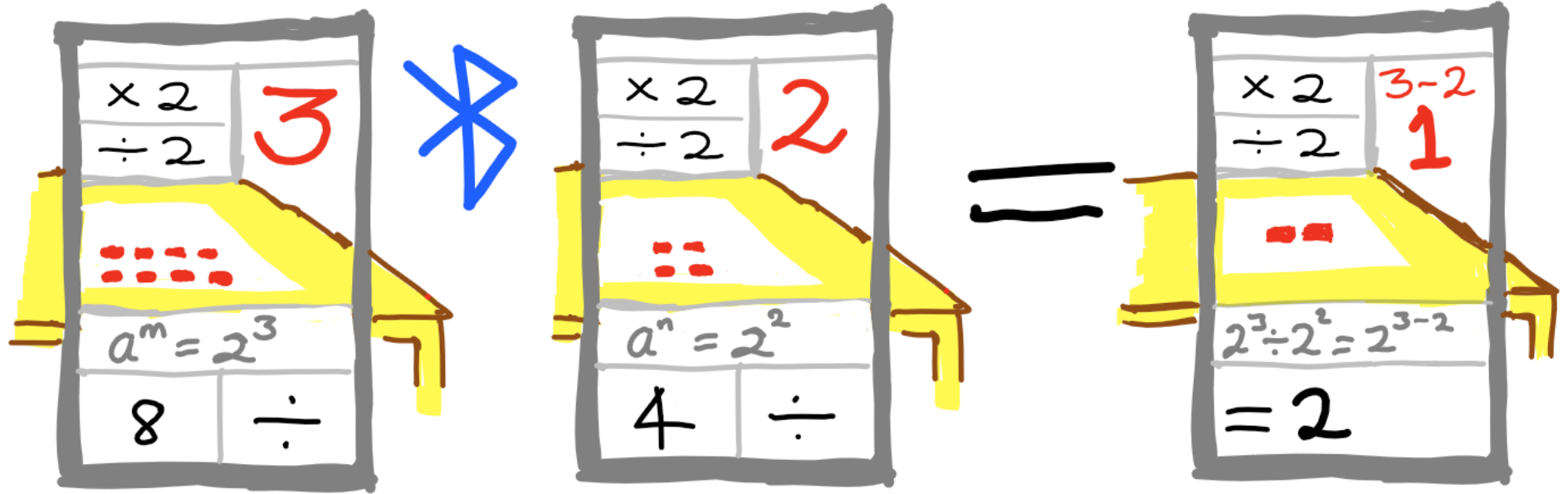
$$1. a^m \times a^n = a^{m+n}$$



Bluetooth pairing

The above screen is shown in both the paired smartphones

2.  $a^m / a^n = a^{m-n}$



Bluetooth pairing

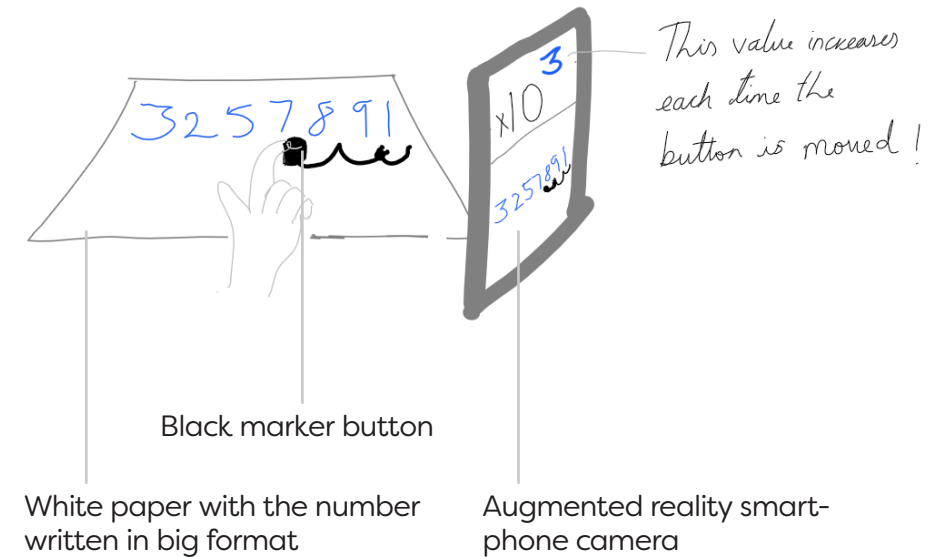
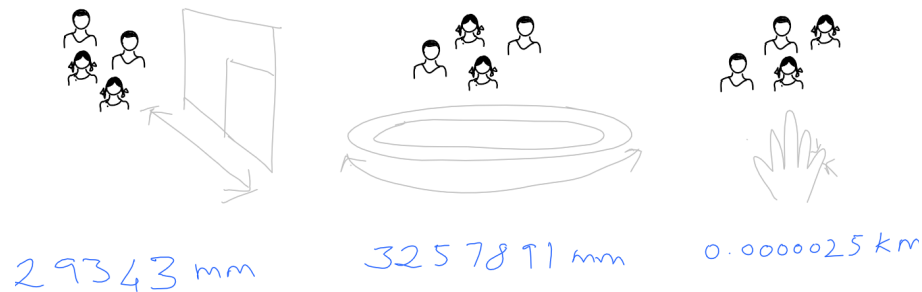
The above screen is shown in both the paired smartphones

What problem and needs does this Idea address?

- Idea of substituting or generalizing numbers into variables is tried to be visualized here.
- The lack of opportunities in the content structure to help the students retain the concepts in memory, is addressed through physical activity and real world visualizations.
- Interactive way of response for an action as a mode of learning and understanding is addressed.
- Even though the application of exponents to diameter of earth or radius of blood cell etc. are relevant, the possibility of applying them to more relatable and day to day objects which are around them is explored through this idea.

### Idea 3 : Class activity with smartphone

1. Divide the class into few groups and assign the following task to each.
2. Make few group to measure the length of classroom, Veranda and ground in meters and covert them to millimeters.
3. Make the other groups to measure small things, or difference in the width of their index finger and ring finger and find the difference in Kilometers.



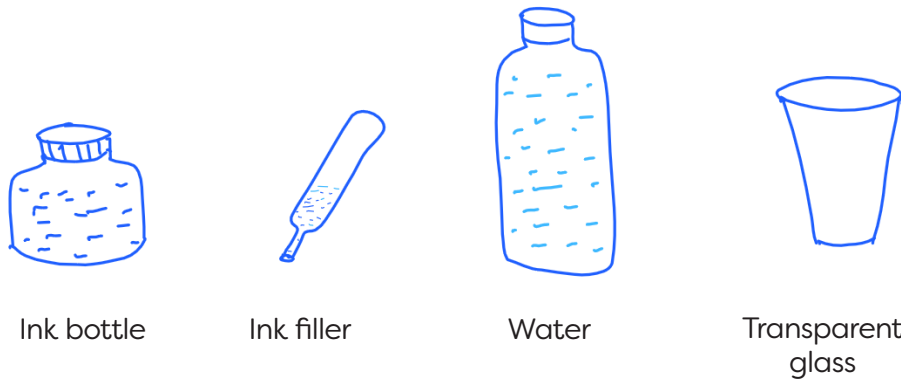
#### What problem and needs does this Idea address?

4. Each group write those values in a white paper in big size with gap between each number.
5. Use a dark colored thing like a black button and use it as the physical decimal point to move in front of the mobile app camera. Each time the button is moved the exponent value of the base 10 changes with respect to the left and right movement.

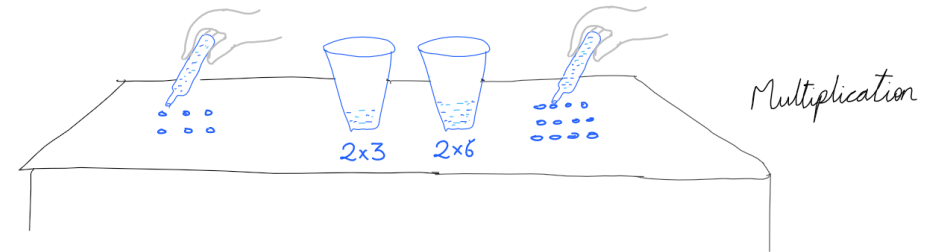
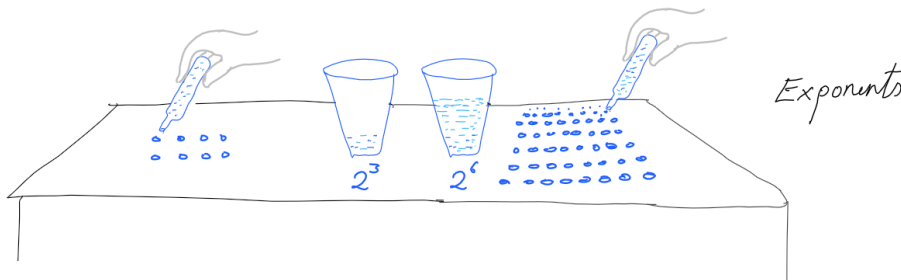
- The need for example activities involving physical or tactile methods which are very less and which could act as a transitional object and help in memory retainment is addressed here.

### Idea 4 : Class activity with ink water-fill glass

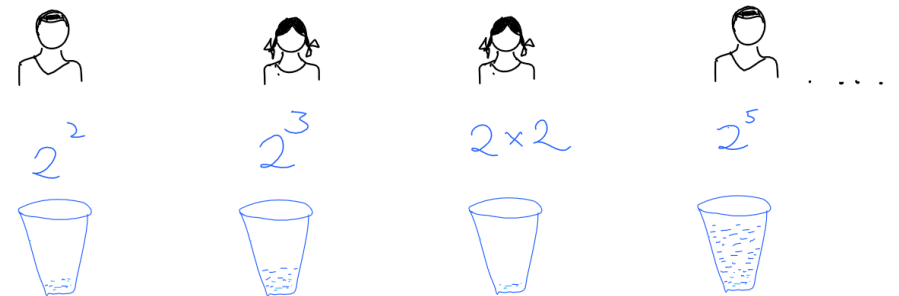
- A class activity with physical objects to understand exponents.
- Each student has to bring an ink filler, transparent plastic glass and some water.
- Teacher assigns squares of numbers to few students, cubes to some, multiplication tables to some and higher exponents to others.



- **Teacher Demonstration** of exponent by comparing with different numbers and also with multiplication.

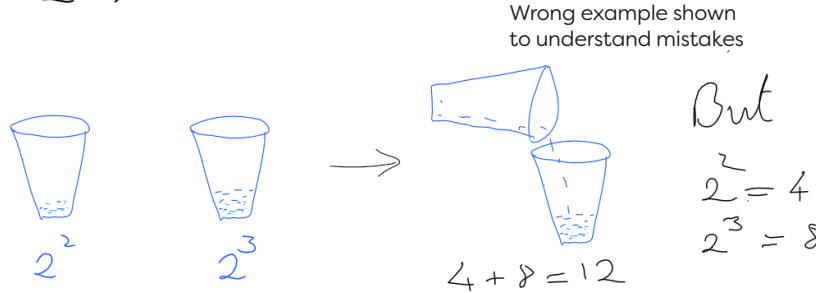


- **Class activity :** Comparison with various example by each student and applying laws of exponents by mixing cups of water to find result.



**Law of multiplication** explained by combining the glasses of water.

Q.  $2^2 \times 2^3 = ?$



But what exponent of base 2 is 12??

This question can be a starting point to introduce decimals and fraction in exponents like  $2^{2.5}$  or  $2^{2/3}$  etc.

The correct way to do it would be to add 4 time the water glass of  $2^3$  together to get the correct answer and compare it with student who filled the glass of  $2^5$ .

$$2^2 \times 2^3 = 4 \times 2^3 = 2^3 + 2^3 + 2^3 + 2^3$$

$$= 8 + 8 + 8 + 8$$

$$= 32$$

$$= 2^5$$

$$= 2^{2+3}$$

$$= a^{m+n}$$

$$2^2 \times 2^3$$

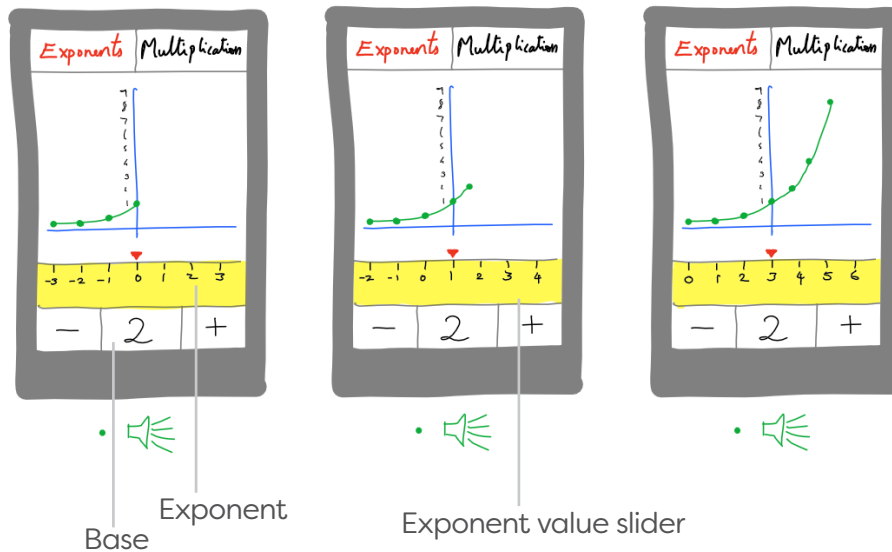
$$a^m \times a^n$$

**What problem and needs does this Idea address?**

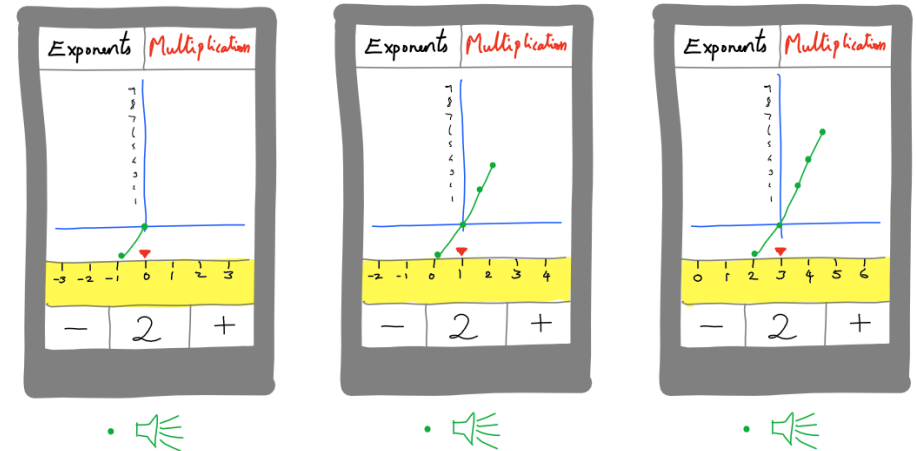
- The lack of opportunities in the content structure to help the students retain the concepts in memory, is addressed here.
- Limitations of the textbook medium to visualize the procedures in the content structure is tried to be delivered through physical activity.

## Idea 5 : Exponential graph & music pitch app

A smartphone application to understand exponents through visual moving graphs and changing music pitch of the notes by comparing with multiplication graph and sound.



Accessibility - Exponential increase in the interval between each music note starting from 2 to 8th octave of music notes. There are 12 notes in each octave and hence a range of 72 notes to show the variation.



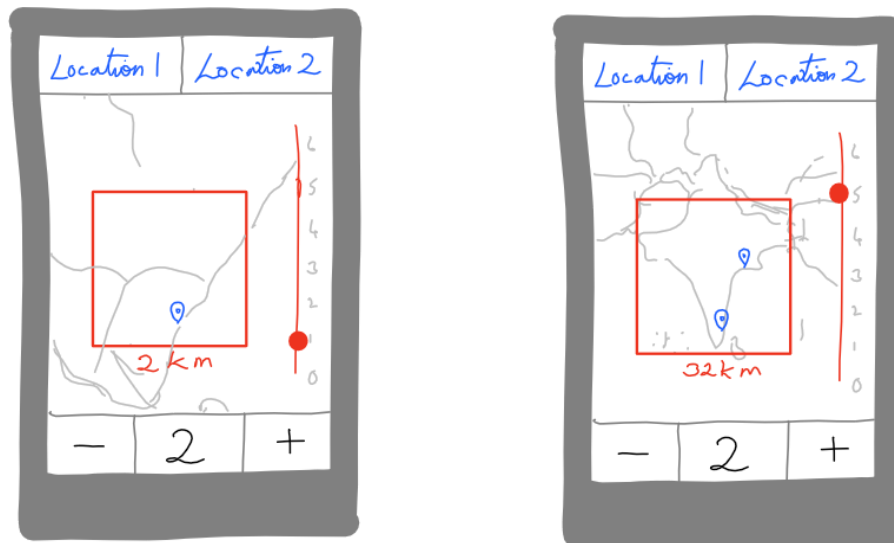
In the multiplication tab, with increase in the exponents value, the graphs increases linearly with linear increase in sound of music notes.

What problem and needs does this Idea address?

- Here sound as a medium to convey the concept is explored to by comparing with exponents and multiplication.
- Interactive methods of learning to understand the concept with various example numbers to reinforce the concept is addressed.

## Idea 6 : Interactive map with exponential zoom

- A google map with zoom features calibrated with respect to the base and exponents.
- App has a square handle to show the zoom level overlaid on the map along with the exponent value like in the Charles Eames video - Power of Ten.
- Here the student can set locations and change the zoom calibration.



- Class activity where students select their home town or any other favorite place and try to accommodate both the location within the smallest exponent of the zoom marker in red as an activity.

### What problem and needs does this Idea address?

- The lack of opportunities in the content structure to help the students retain the concepts in memory, other than repeated practice is addressed.
- Interactive way of response for an action as a mode of learning and understanding is explored here.

## Idea 7 : Social science and math teacher together activity

Both the Social science teacher and Math teacher conduct the class together.

**Social science teacher** : In the year 1939, world war 2 had begun. The European nations of Britain and France declared war against Germany for their invasion into Poland.



**Math teacher** : ..... year ..... Perfect square..... Exponents intro.....

**Social science teacher** : ..... Alan Turing... Enigma machine.....

**Math teacher** : ..... Large number to small .....Scientific notation.....

**Social science teacher** : ..... End of war ..... Atom bomb in Hiroshima and Nagasaki.....

**Math teacher** : ..... Exponential destruction due nuclear fission reaction.....

**What problem and needs does this Idea address?**

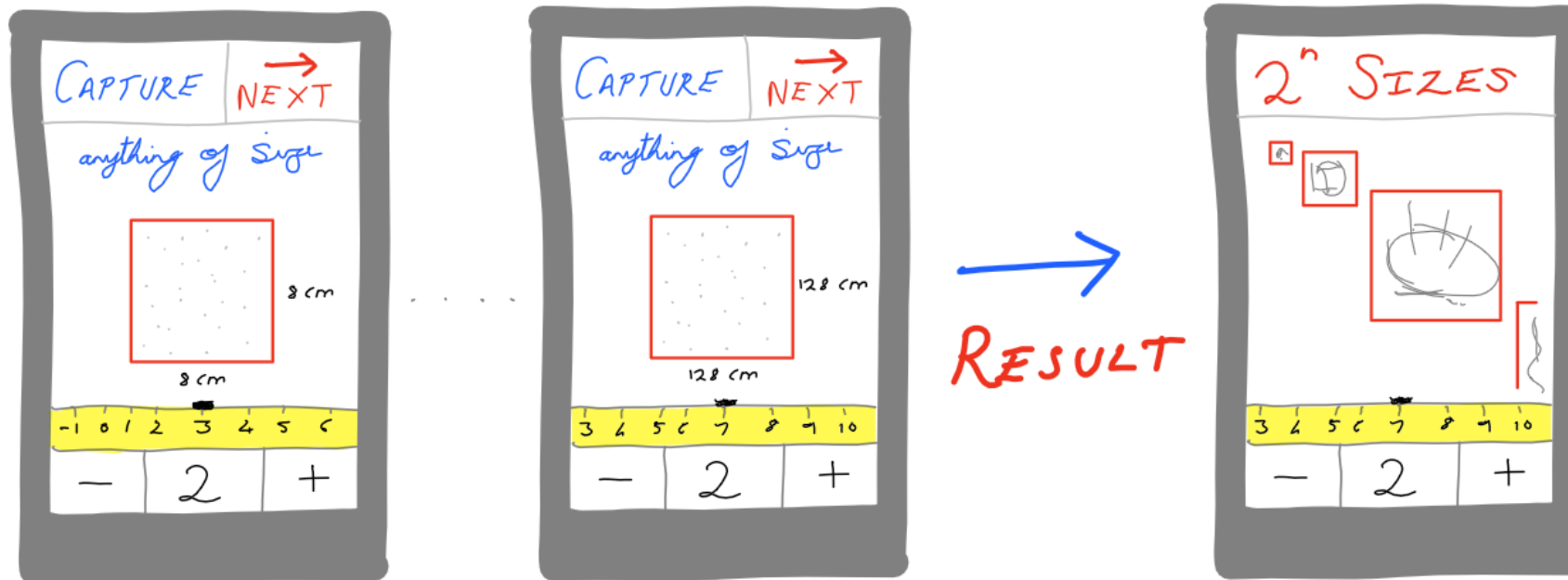
- The problem of math being a subject with less emotions or memories to associate with, while learning the concepts is tried to be bridged with the help of other subjects like history.

## Idea 8 : Exponential size compare app

Take photo of objects with the smartphone camera app showing overlaid square with cm value of the selected base and exponent value. After proceeding with next button for th at sequence the app will compile all the images and compare them with respect to size. The presents the images in ratio of their size to visually understand the exponential change in size.

What problem and needs does this Idea address?

- The lack of opportunities in the content structure to help the students retain the concepts in memory, is addressed through physical activity involving day today objects around them.



## 7. Redefined Goal

---

From the above design ideas discussed, the following were the criteria to choose a final idea from them -

- Physical activity involving real world objects, to connect the exponents concept with a memory.
- Visual representation of the concept with real world scenarios and objects.
- Interactive way of response for an action as a mode of learning and understanding.
- Transitional objects as a bridge between the virtual and physical world to explore the possibilities of learning mathematics.
- Collaborate other subjects like history and geography with mathematics concepts to evoke emotions and connect memories with the concepts.
- Course structure with repeated practice and learning from the mistakes.
- Sound as a medium to convey the concept.

As per the criteria, the Idea 2 had most of them fulfilled and was chosen to be developed as the final product. The refined goal of the project was to satisfy the criteria of making the 7th standard children understand the concept of exponents and powers by making them do physical activities with day to day objects found with them. This acts as a tool for retaining the concept in memory and have a better recall. The goal was also to visualize the facts and procedures with augmented real world scenarios which acts as a guide to better apply those concepts in similar situations.

With respect to the Bloom's taxonomy [5] the project goal was narrowed to make the students remember, understand and apply the facts, procedures and concepts and not beyond this capacity for 7th students.

The focus was also more towards the idea of "Michael Eisenberg, Mindstuff: educational technology beyond the computer" [7] where Emotional affordance is provided through the Software applications to augment use of tangible material.

## 8. Final design & prototype

The process started with rough wire-frames of the augmented reality application and then defining the flow of content and activities. After trying few mock-up design layouts the working prototype was developed for evaluation.

### Wire-frames

The basic content of the application was defined here which are -

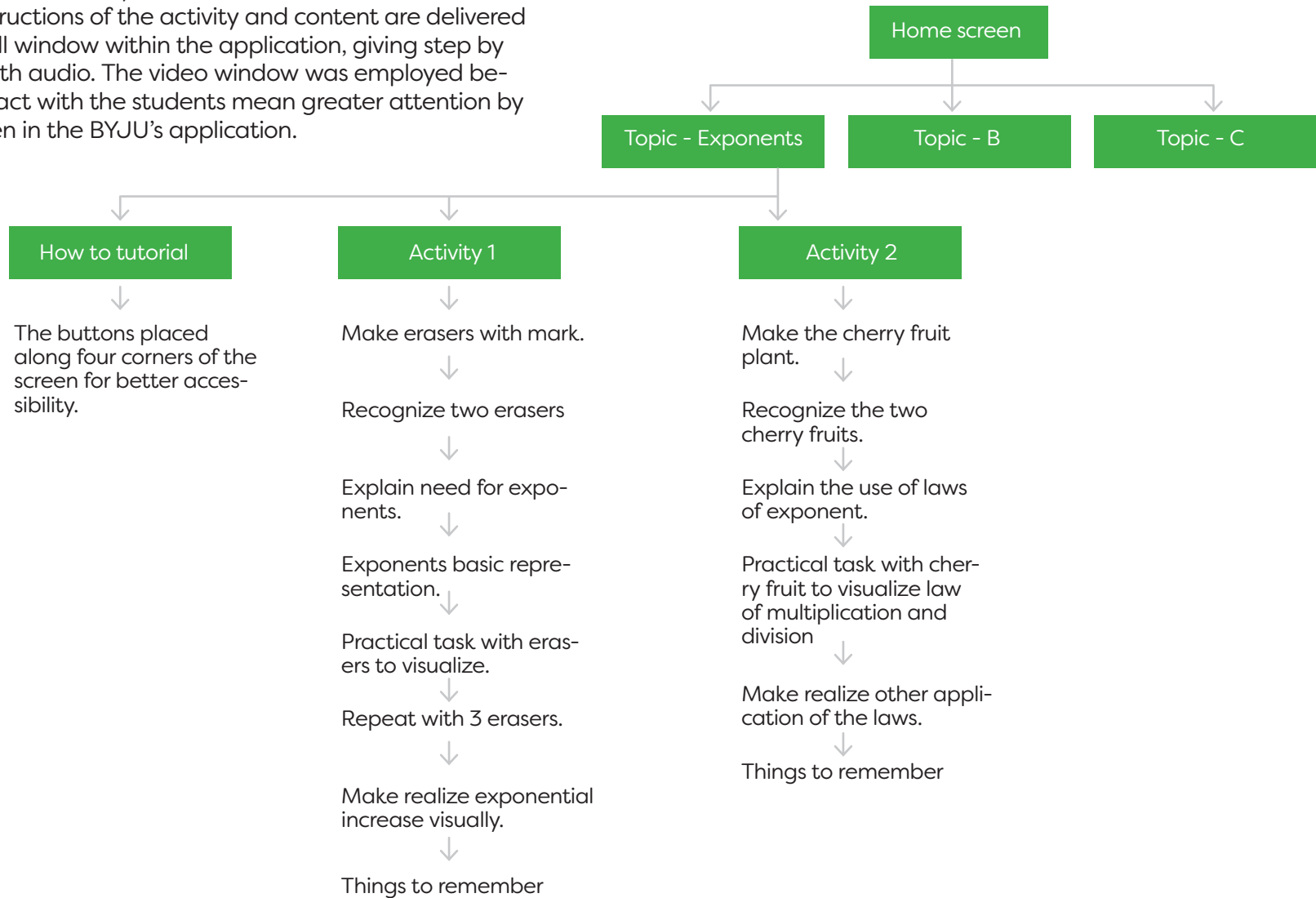
- Home screen to navigate to different topics.
- Activity 1 to teach the basics of exponents with the eraser activity.
- Activity 2 to teach the laws of exponents using the cherry plant activity.

All the component of the screens were defined in this section.



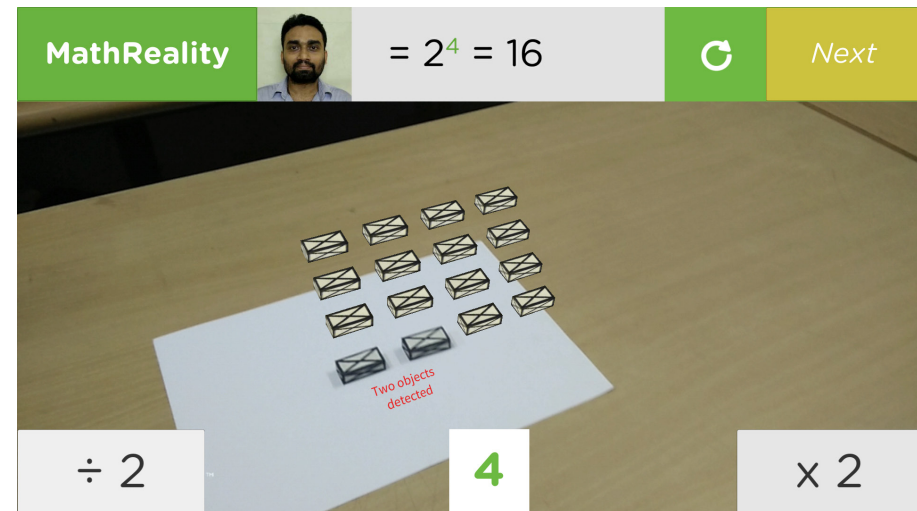
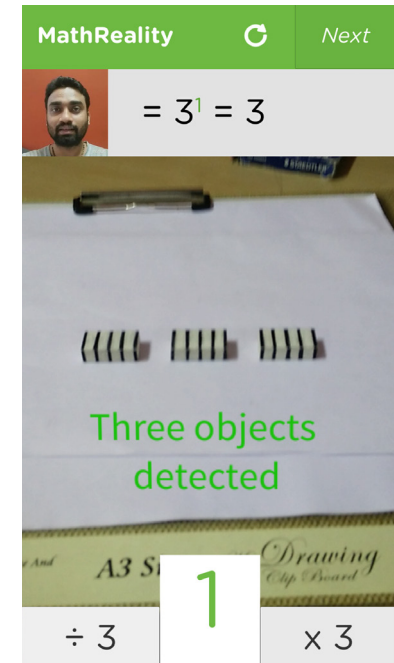
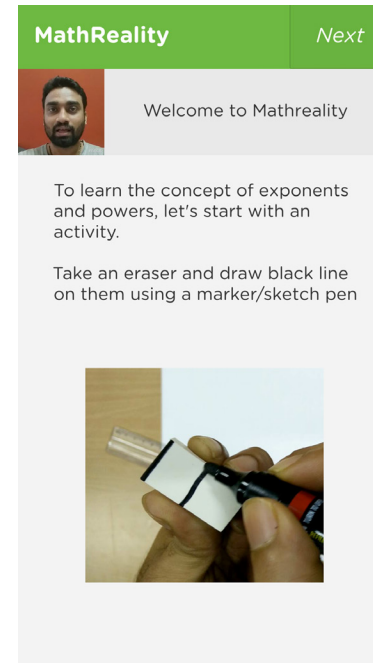
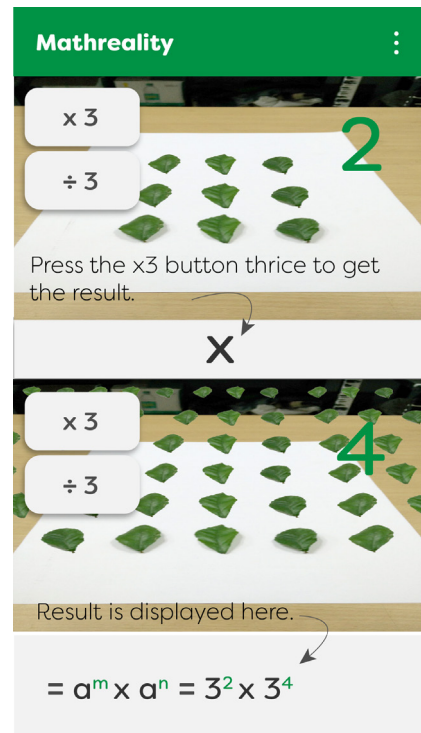
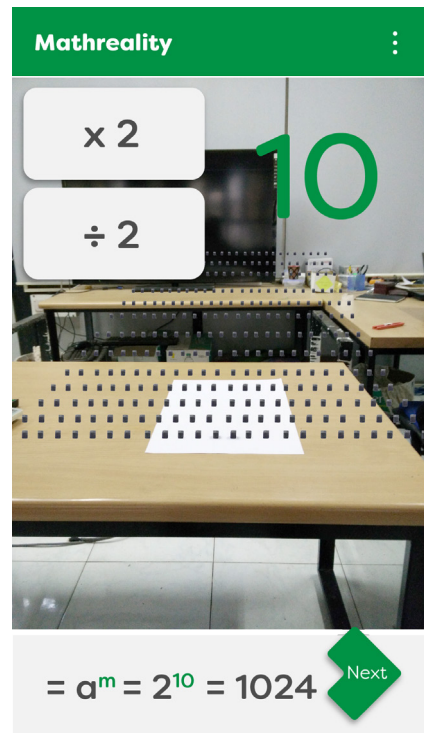
### Flow diagram of the application

The android application mainly uses touch interactions on smart-phone and the instructions of the activity and content are delivered by a tutor in a small window within the application, giving step by step instructions with audio. The video window was employed because, an eye contact with the students mean greater attention by the students as seen in the BYJU’s application.



### Visual design mock-up layouts

The layouts were started off in portrait mode but later switched to landscape mode for better view-ability of the augmented objects. The important control buttons namely the “next”, “menu” and control buttons were placed along the corners of the screen for better accessibility incorporating the design of EDGE [24].

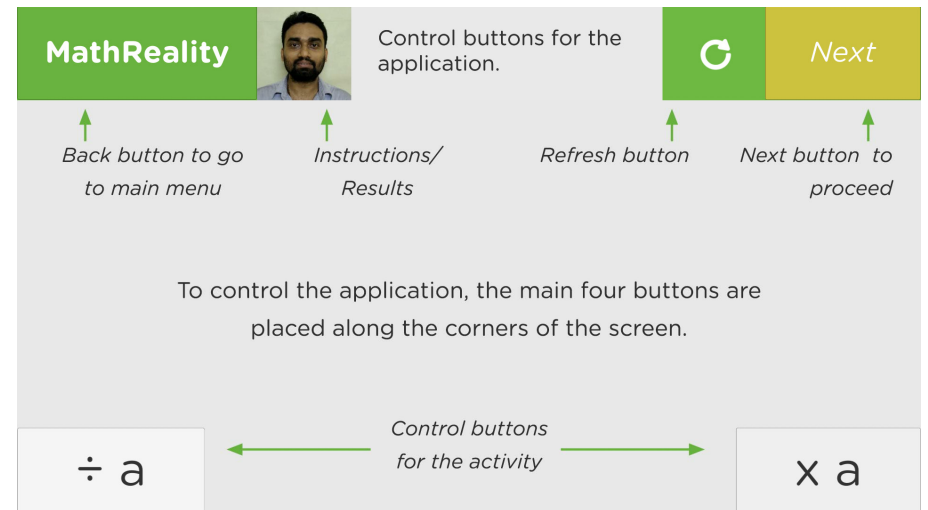
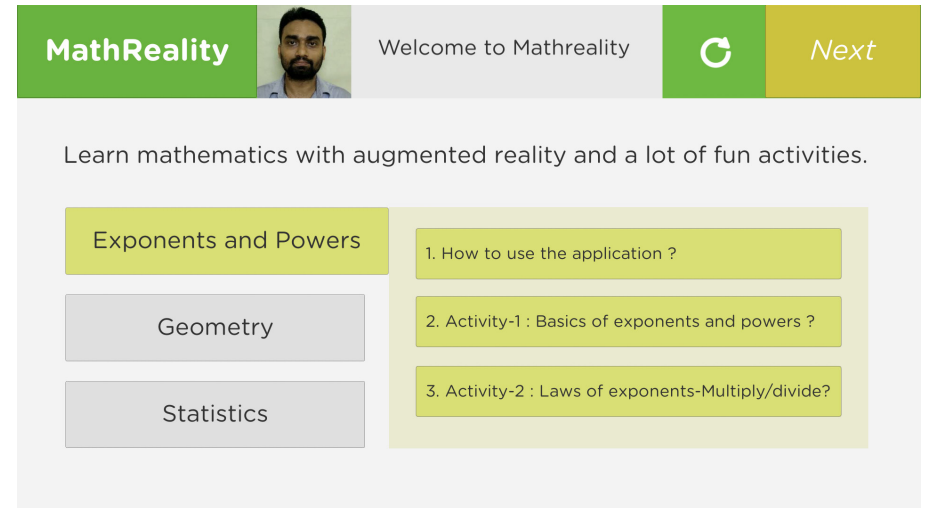
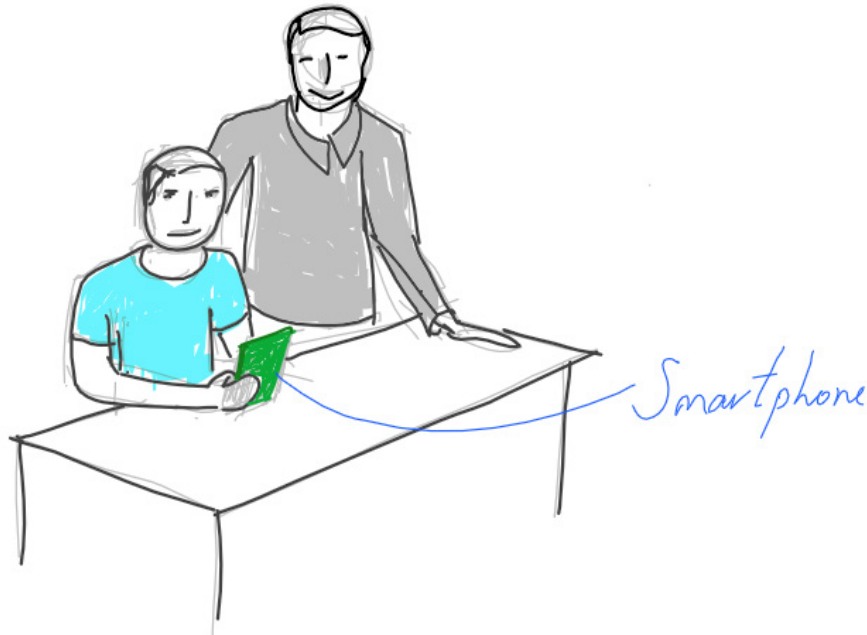


### Final design prototype in a scenario

Lets consider a scenario where this augmented reality application is used by “Shyam” in his father’s smart-phone under the guidance of his father in home.

#### 1. Home screen and topic selection

- Shyam hears the instructions of the tutor from the small video window as the application starts and selects the next button to proceed.
- He understands the control buttons of the application in the “How to use” section and proceeds next.





2. Activity 1 with erasers to learn basic concept of exponents.

MathReality  Activity 1 to learn basics of exponents & powers  Next

Today we are going to learn the concept of exponents and powers. let's start with an activity.

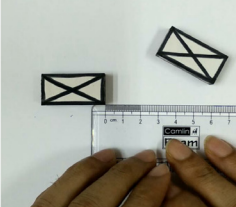
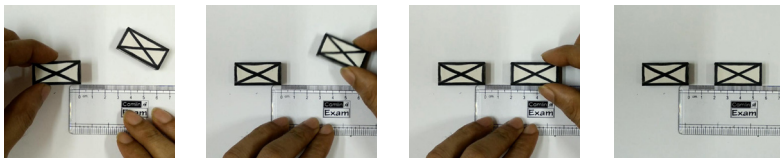
Take an eraser and draw black line on them using a marker/sketch pen as shown.






MathReality  Setup the erasers as shown below  Next

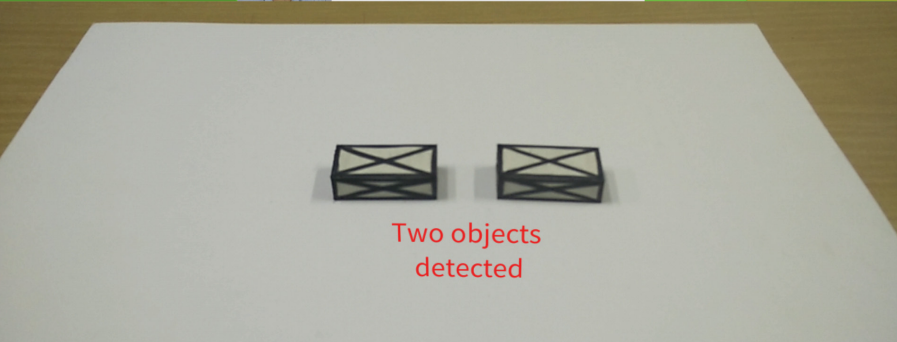
Now make one more eraser with the same black lines and place them on a white paper with 2cm gap between them, as shown.

The next step is to recognize the two erasers by your smartphone.

- Shyam follows the instructions to do the first activity to understand the basic concept of exponents.
- He takes an eraser and marks them as per the instruction and repeats them to make two similar erasers.
- He proceeds next and as per the instruction brings the smartphone close to the aligned eraser to recognize them with “Two objects detected” feedback text augmented in the real world.
- This forms the first small chunk of this task with a congratulations message to motivate Shyam.

MathReality  **Congratulations!** The erasers you made is recognised. Proceed to next.  Next




Two objects detected

### 3. Theory on why exponents and powers is needed.

- Shyam is made to imagine a scenario where if each day the erasers marked by him is double the number, how many he would have made in the 8th day as a question.
- Then he is explained the use of representing repeated multiplication of numbers as exponents.
- Next he is asked to bring the smartphone closer to the erasers marked to visualize the number of erasers he would have made on the 8th day.

MathReality




Exponents and powers  
basics of representation.

↻

Next


As per our case, the repeated multiplication of 2 for 8 times is represented in exponents and powers as follows -



Here 2 is called base and 8 is called the exponent and the expression is called as “2 to the power of 8”.

Now let’s visualize the result with each increment in the number of exponents, by detecting the eraser and multiplying by 2. Notice the magnitude of exponential increase visually on the number of erasers you would have made on the 8th day from today.

MathReality



Difficulty in representing  
repeated multiplication.

↻

Next

Assume that each day from now if the number of eraser made by you like this is doubled, how many erasers will you make on the 8th day?


The answer is :

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

This is indeed a tedious way of representing multiplication of the same number.

Hence let’s learn the concept of exponents and powers to represent multiplication of repeated numbers in an easier way next.

MathReality



Bring your smartphone closer to recognize the eraser and press “x2” button 8 times to visualize and then try “÷2” button.

↻

Next



÷ a


1

x a



#### 4. Visualization of erasers by augmenting in real world

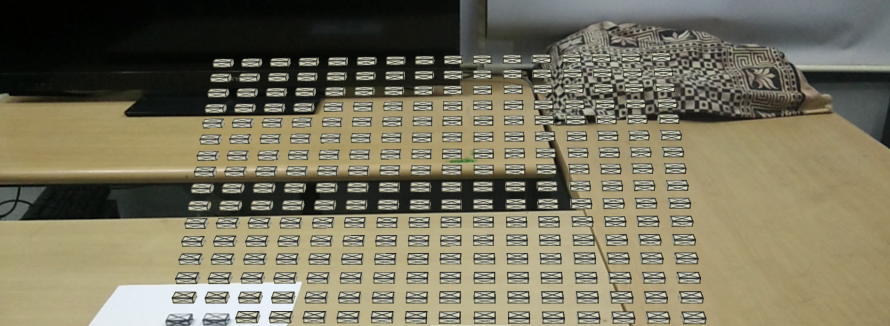
- Shyam touches the “x2” button to augment the erasers and visualize how the erasers are doubled for each press.
- The result is displayed on the top and the augmented erasers give a visual key about how much space these many erasers cover in the real world if they were made.
- He also tries the “÷2” button to view the erasers divided by 2 and learn that if we keep dividing base, the exponents are represented in negative numbers and are called negative exponents.
- He now makes one more eraser with the same mark to make the base “3” and see the difference in visualization of erasers.

MathReality  =  $2^{-1} = 1/2$   Next





÷ 2 **-1** × 2

MathReality  =  $2^8 = 256$   Next



÷ 2 **8** × 2

MathReality  Exponent and powers with higher base and understand exponential increase.  Next

Now while pressing the divide button lesser than “0”, you must have noticed that the exponents become negative in power and gets divided into pieces exponentially. Hence very small numbers can be expressed through negative exponents.

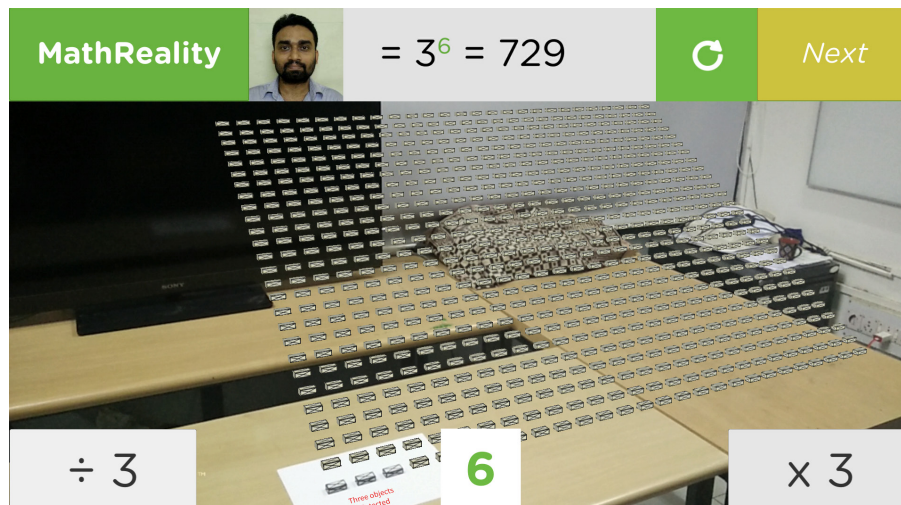
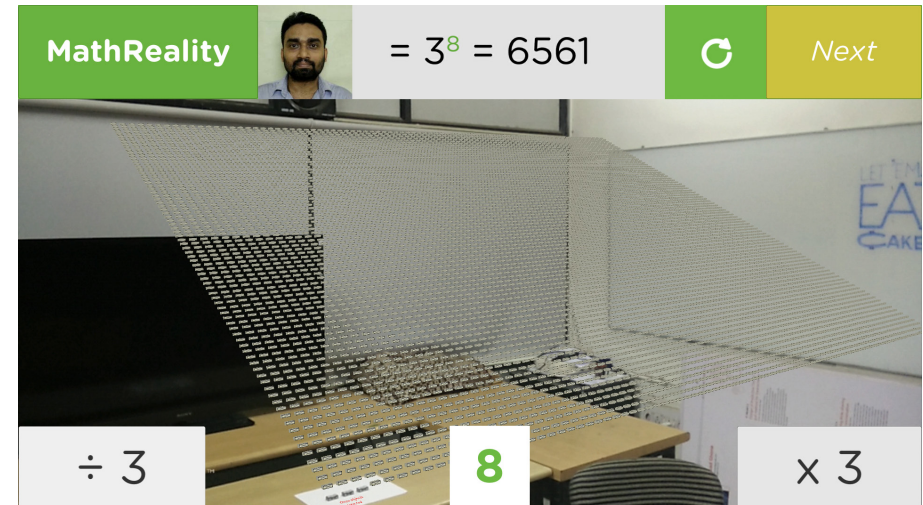
eraser which makes the base “3” and visualize them with increasing positive as well as negative exponents.

Notice in the next screen that the increase in the number of eraser grows very rapidly and hence called exponential increase.

Similar to the two erasers visualized now, let’s make another

### 5. Augmentation with different base and things to remember

- Shyam makes one more eraser and visualizes the 3 marked erasers by following the same previous procedure.
- He notices that with increase in base the exponential increase of the erasers made each day increases very rapidly filling up the entire room.
- For each increase or decrease in the exponents, the application also gives audio feedback sounds which are exponential in nature.
- This exercise concludes the basics of exponents with things to remember points summarized.



**Things to remember :**

1. Repeated numbers multiplied are represented as exponents over the base value.
2. If you divide repeatedly below zero exponent, it is called negative exponents.
3. Exponential increase or decrease is very rapid with increase in base.



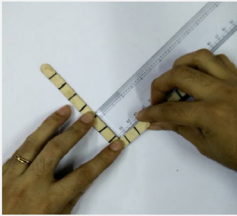
Now lets move to the next task and learn the laws of exponents namely,

1. Law of Multiplication of exponents
2. Law of Division of exponents.

## 6. Activity 1 with erasers to learn basic concept of exponents.



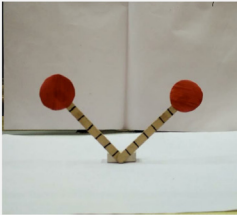
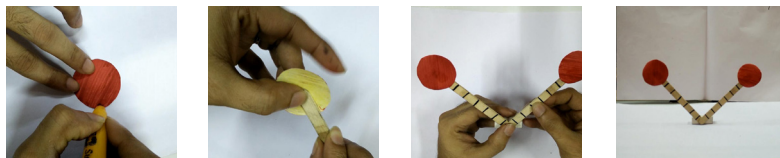
MathReality Activity 2 : Laws of exponents and powers

Lets start this activity by collecting 2 ice-cream sticks, a white paper and color sketch pens. Draw lines over the ice-cream stick as show below and stick them together in 90 degree angle and proceed next.


MathReality Activity 2 : Laws of exponents and powers

Now make 2 circular paper cutout of 2cm radius and color them with red sketch pen as shown below. Stick these two circular papers to the tip of the ice-cream stick with the circle's center coinciding the sticks tip. Stick this entire thing on an eraser base as shown. Now this represents 2 cherry fruits in a cherry plant.

- Shyam follows the instructions to do the second activity to understand the basic concept of exponents.
- He marks the ice-cream stick as per the instructions and sticks them with red circular paper cut-outs to a base representing a cherry plant with 2 cherry fruits on them.
- After making the cherry plants, the laws of exponents is explained with formulas and with respect to this particular example so that they could be visualized in augmented reality.
- Next the smart-phone is brought closer to the cherry fruits to visualize them grow, when two exponents are multiplied or divided.

MathReality Laws of exponents - Multiplication and division

Lets imagine that each day the plant bear new cherry fruits exponentially with the base now being 2. As learnt from the previous activity, on **2nd day** there will be  $2^2 = 4$  new cherry fruits in the plant and on the **3rd day** it will have  $2^3 = 8$  new cherry fruits in the plant.

Now to multiply the number of new cherry fruits on the 2nd and 3rd day, we apply the law of exponents of multiplication which is  $a^m \times a^n = a^{m+n}$ . In our case  $2^2 \times 2^3 = 2^{2+3}$ . Similarly for division  $a^m \div a^n = a^{m-n}$ . Now lets proceed to visualize this in real world.

### 7. Augmentation with different base and things to remember

- Shyam grows the cherry plant in the real world, with each day the number of new cherry fruits getting doubled.
- He selects the number of new cherry fruits grown on the 3rd and 4th day to multiply them and visualize the growth of cherry plant and see the result, which is the number of new cherry fruits on the 7th day.
- He also tries it with divide button to visualize the law of division of exponents.
- He also learns about how to calculate the total number of cherry fruits in the cherry plant in a given day.
- This concludes the activity 2 and proceeds to the next. This project could be expanded to other activities to teach the other concepts.

**Things to remember-**

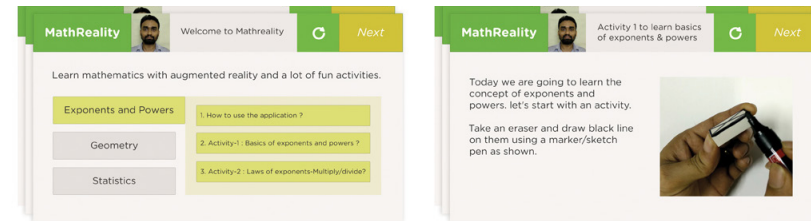
1. If you multiply 2 powers with same base, the resultant exponent is the addition of exponents values.
2. If you divide two powers with same base, the resultant exponent is the subtraction of exponent values.

**Note:** To calculate the total number of cherry fruits on the 5th day for example, as visualized in the previous step it will be  $2^1 + 2^2 + 2^3 + 2^4 + 2^5 = 62$  fruits totally on the cherry plant.

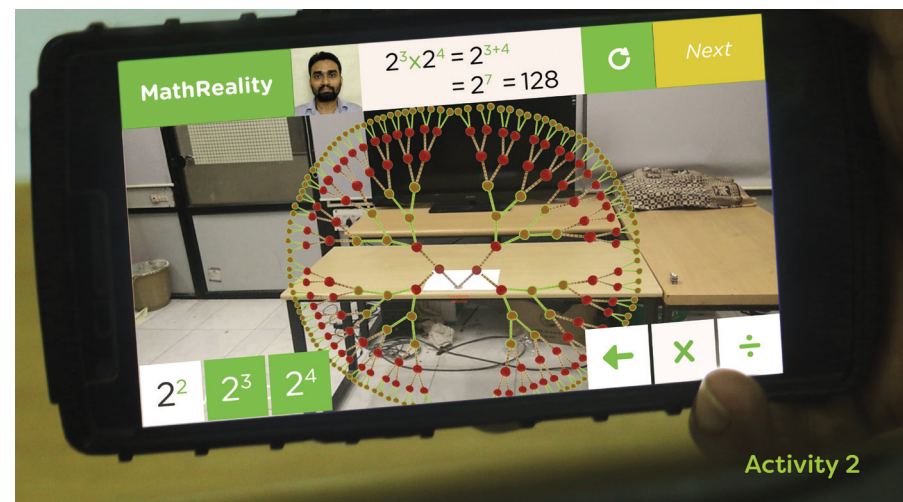
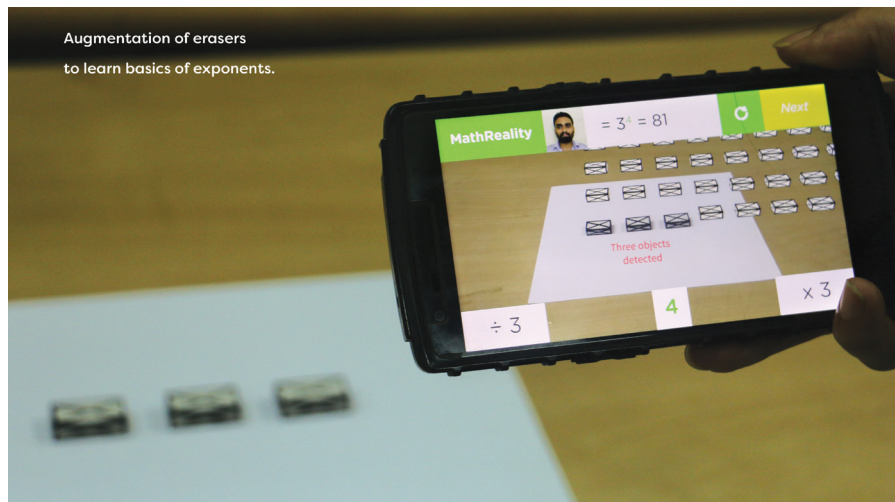
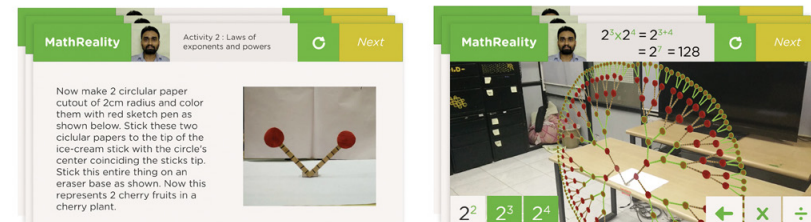
Now lets proceed to the next activity to learn the law for paranthesis.

## Working prototype

The working prototype is made using the software called UNITY with Vuforia plugin. C# coding is used to write the logic for the application. The application can recognize objects and its count and augment them with respect to the exponents incremented through the application. The following are the screen-shots of the working application to proceed for evaluation with the students.



Screenshots of the high fidelity working prototype



## 9. Evaluation

The solution is evaluated for its effectiveness to make the students gain cognitive abilities as per the following components of M David Merrill's performance-content matrix :

1. Remember-fact
2. Remember-concept
3. Use-concept
4. Remember-procedure
5. Use-procedure.

To assess these components, Kirkpatrick's Four-Level Training Evaluation Model is also used to frame the methods of conducting the evaluation. The Four-level model is as follows-

**Level 1: Reaction** - The degree to which students find the training favorable, engaging and relevant.

**Level 2: Learning** -The degree to which students acquire the intended knowledge, skills, attitude, confidence and commitment based on their participation in the training.

**Level 3: Behavior** -The degree to which students apply what they learned during training when they are exposed to the real world problems.

**Level 4: Results** - The degree to which targeted outcomes occur as a result of the training.

The prerequisite assumption for evaluation is that the students have been taught the topic of exponents once in their school.

From this 4 levels, Level 2 of evaluating the learning is conducted by a pretest and post-test for the augmented reality application where the questions asked in them are to assess the 5 components of the performance-content matrix as discussed before.

Level 1 is evaluated by directly asking feedback from the students after the level 2 evaluation task. Questions like "did you like the product" in likert scale and "how easy the activity was?" etc were asked.

To conduct the pretest and post-test, a set of 20 questions were prepared and to each student a set of random 6 questions were asked and within this 6 questions all the 5 components of the performance-content matrix were included. The improvements in mark for each type of question were assessed to measure the effectiveness of the augmented reality application.

Following are the 20 questions with their component types :

- |   |                           |
|---|---------------------------|
| 1. When you repeatedly multiply the number 3, for 6 times what is the answer?                                   | <i>Remember<br/>-fact</i> |
| <i>Answer : 729</i>   | <i>5 marks</i>            |
| 2. How will you represent $3 \times 3 \times 3 \times 3 \times 3 \times 3$ in a smaller form of representation? | <i>Remember<br/>-fact</i> |
| <i>Answer : <math>3^6</math></i>  | <i>5 marks</i>            |

3. When you repeatedly multiply number 2, for 8 times what is the answer?

**Answer :** 256

Remember  
-fact

5 marks

4. How will you represent  $1 / 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$  in a smaller form of representation?

**Answer :**  $2^{-8}$

Remember  
-fact

5 marks

5. Which one is greater  $2^8$  or  $3^6$ . Answer without calculating the result.

**Answer :**  $3^6$

Remember  
-concept

10 marks

6. Which one is smaller  $2^{-32}$  or  $5^{-25}$ . Answer without calculating the result.

**Answer :**  $5^{-25}$

Remember  
-concept

10 marks

7. What is  $3^{14} \times 3^{20}$ . Express the result in exponential form.

**Answer :**  $3^{(14+20)} = 3^{34}$

Remember  
-concept

10 marks

8. What is  $4^{-4} / 4^{20}$ . Express the result in exponential form.

**Answer :**  $4^{(-4-20)} = 4^{-24}$

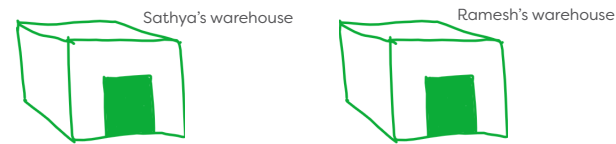
Remember  
-concept

10 marks

9. There are  $5^{12}$  mangoes with farmer Sathya and  $4^{12}$  mangoes with farmer Ramesh. Lets consider they have to be stored in the government warehouse as below. Which warehouse will fill first? Sathya's warehouse or Ramesh's warehouse?

Use-concept

15 marks



**Answer :** Sathya's warehouse

10. Kavitha and Archana are pregnant and are going to have babies soon. A baby develop inside them when a single cell of the baby multiplies exponentially with each day and the number of cells divides itself to form new cells. Now Kavitha's baby has developed into  $2^{60}$  cells and Archana's baby has developed into  $2^{90}$  cells. How many months old are the babies now inside Kavitha and Archana?

Use-concept

15 marks

**Answer :** Kavitha's baby - 2 months old and Archana's baby - 3 months old

11. There are two fruits basket on table. In one fruit basket there are 3 apples in them. In another basket there are  $(\text{no. of apple})^2$  number of oranges comparing apple basket. How many oranges does the 2nd basket have?

Use-concept

15 marks

**Answer :**  $3^2 = 9$ . There are 9 oranges in the 2nd basket.

12. Meena's father had  $2^3$  apps in his smartphone and Raju's father had  $2^5$  apps in his smartphone. Shankar's father had a multiplication of the number of apps with Meena's and Raju's father. How many apps did Shankar's father have? Express the answer in exponents.

**Answer :**  $2^3 \times 2^5 = 2^8$  apps

13. If  $a = 2$ ,  $m=10$  and  $n=14$ , what is  $a^m \times a^n$ ? Write the answer with steps.

**Answer :**  $a^m \times a^n = a^{m+n}$   
 $2^{10} \times 2^{14} = 2^{10+14}$   
 $= 2^{24}$

14. If  $a = 3$ ,  $m=8$  and  $n=13$ , what is  $a^m \div a^n$ ? Write the answer with steps.

**Answer :**  $a^m \div a^n = a^{m-n}$   
 $3^8 \div 3^{13} = 2^{8-13}$   
 $= 2^{-5}$

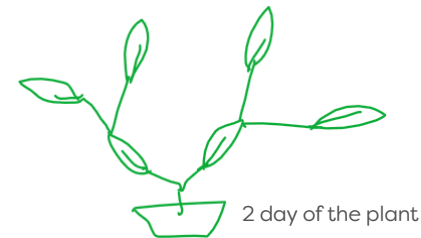
15. What is  $2^{10} \times 3^{14}$ . Express the result in exponential form.

**Answer :** They cannot be expressed exponentially because the base of the powers are different.

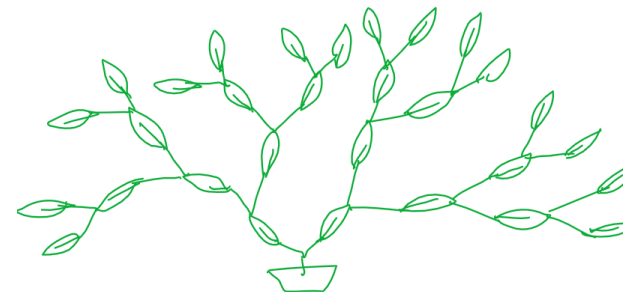
16. What is  $5^8 \div 3^4$ . Express the result in exponential form.

**Answer :** They cannot be expressed exponentially because the base of the powers are different.

17. There are 2 panda bears in your backyard along with a plant with leaves for them to eat. Now there are 2 leaves on the plant. If each day the number of new leaves grown is double the previous day, How many leaves will be there on the 5th day from now? Answer by completing the plant visually.



**Answer :**



*Remember  
-procedure*

10 marks

*Use-procedure*

15 marks

*Use-concept*

15 marks

*Remember  
-procedure*

10 marks

*Remember  
-procedure*

10 marks

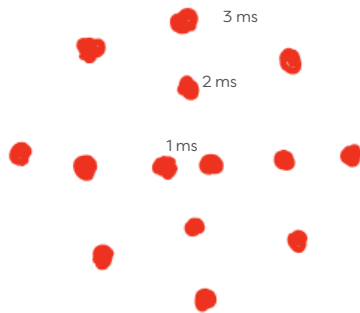
*Remember  
-procedure*

10 marks

18. From the question 17 for the two panda bears, if we distribute the leaves to eat such that each get “a” number of leaves, where “a” is equal to the number of new leaves on 5th day divided by number of new leaves on second day. How many leaves does each panda get to eat?

*Answer :  $a = 2^5 / 2^2 = 2^{5-2} = 2^3 = 8$ . Hence each panda gets 8 leaves to eat.*

19. In the world war 2, when the nuclear bomb exploded, each millisecond the number of fire particles doubled in number. If there were these many fire particles at the 3rd millisecond, how many particles would have to there in 1 sec. Express the answer in exponential form.



*Answer : In 1 millisecond there are 2 fire particles hence the base is 2. At 3rd millisecond number of fire particle is  $8 = 2^3$ . Hence at 1 sec we will have  $2^{1000}$  fire particles. 1sec = 1000 millisecond.*

Use-procedure

15 marks

Use-procedure

15 marks

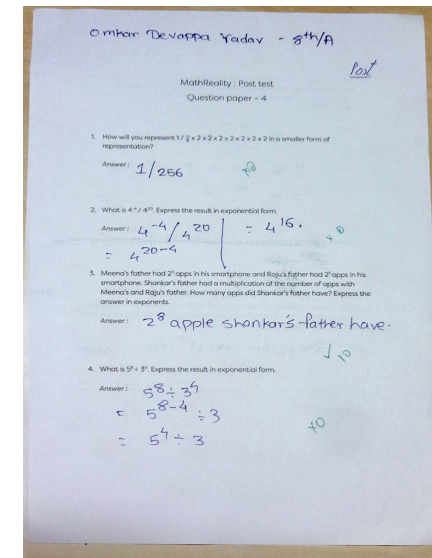
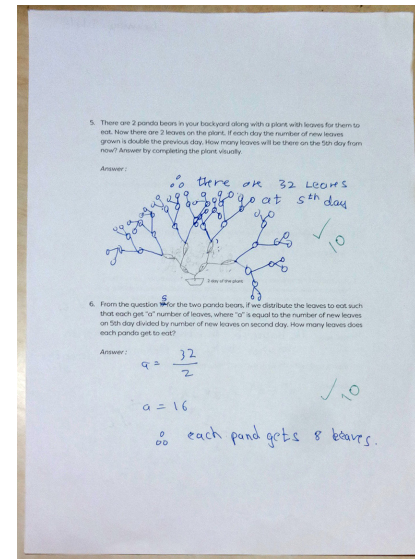
20. There was cake on the table which was made of  $2^{24}$  pieces. It was surrounded by  $2^8$  to ants to eat them fully. The leader of the ants decided to divided the cake equally for each ant. How many cake pieces will each ant get?

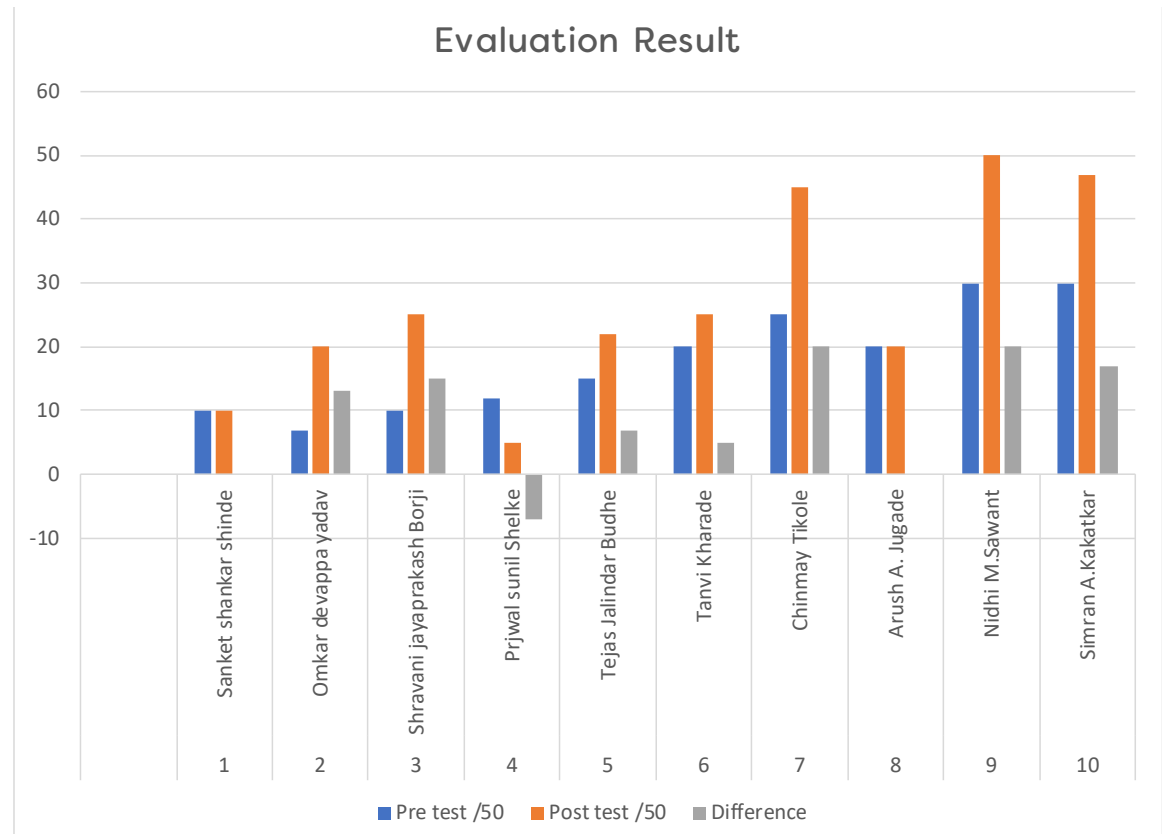
*Answer :  $2^{24} / 2^8 = 2^{24-8} = 2^{16}$ . Hence each ant can eat  $2^{16}$  bread pieces each.*

Use-procedure

15 marks

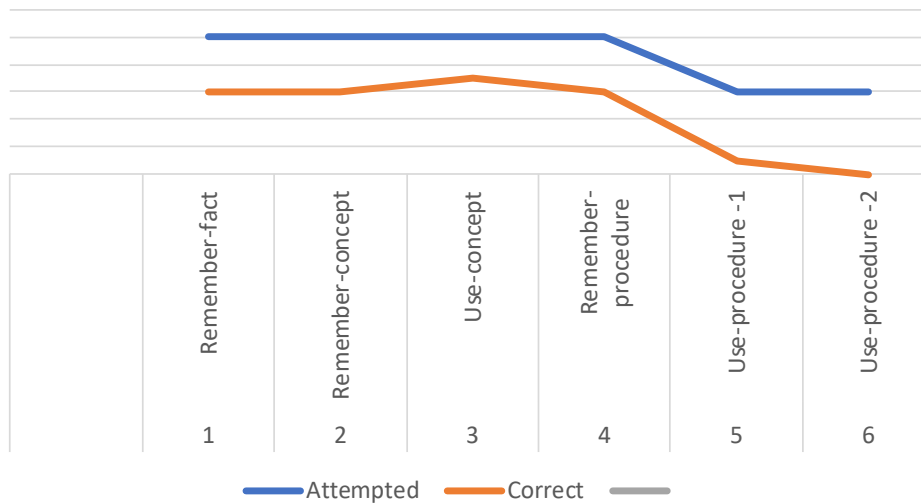
Each component has four questions each and while conducting the pretest and post-test, 6 questions from each component were picked randomly to ask the students.



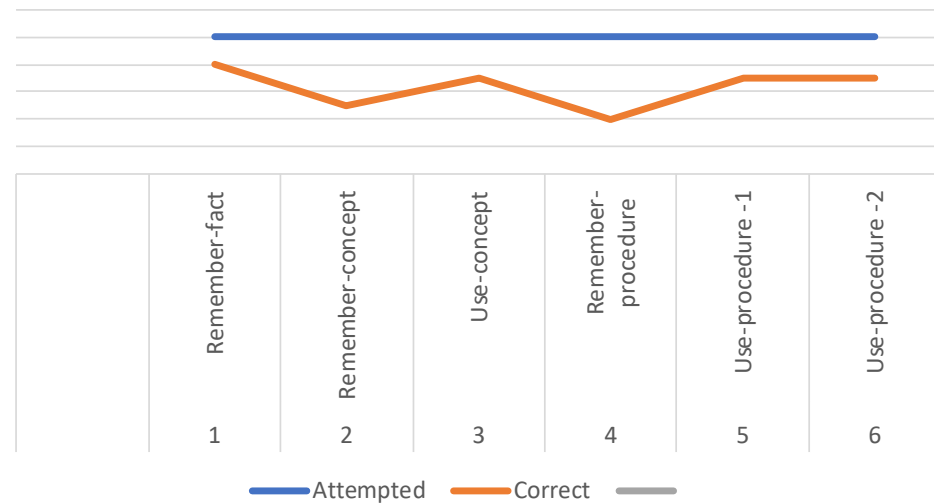


- 10 students were made to do the pre-test and were proceeded to use the application individually. After that a post test was conducted on the same 10 students for evaluating their performance.
- The performance was compared in two ways. One by comparing the overall change in the total marks scored by each student from pre-test to post test. Secondly by comparing the number of questions attempted and answered correctly across both tests with respect to the component classification of the questions.
- In the first comparison result, the change in overall marks were positive with an **average of 9 marks increase** from pretest to post test with a minimum change of -7 marks and maximum increase of 20 marks.
- The average score of the students increased from 17.9/50 to 26.9/50 with the highest score possible being 50 marks.

Students count on questions type - Pre test



Students count on question type - Post test



- In the second comparison result, the pre test questions based on Use-Procedure component were not even attempted by the students and the number of students with correct answers took a declining curve when moving from Remember-fact component to Use-Procedure component.
- After using the augmented reality application and doing the post test, the question based on Use-Procedure were at least attempted by all the students and the number of students with correct answers also increased while moving from Remember-fact component to Use-Procedure component.

- Hence from the pre test and post test evaluation it is evident that students who feared to even attempt the tough questions, at least tried it out after using the augmented reality application.
- There was a considerable increase in the correct answers while moving from the first to last component type of the questions to be answered.

## 10. Feedback and Conclusion

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The general feedback from the students and teachers were as follows:

- The augmented reality duplication was received well with wow factor involved when the erasers fill the room or when the cherry tree multiplies and grows in size.
- The UI needs to be more consistent as the students had difficulty in using the actions learned from the first activity into the second activity.
- The audio instructions with tutor on display in small window was a positive factor to grab the attention of students, but they felt bored to go through the entire textual content.
- The teacher had a comment that even though there is an initial wow factor, it is difficult for that effect to get translated into marks without repeated practice.
- The opportunity to apply the learned concept through these activities, in real world scenarios need to be addressed, as currently it is restricted into the written test with application oriented questions.
- How can these methods be applied to other abstract mathematics concepts needs to be explored.

### **Conclusion :**

The augmented reality application can have a positive impact on the ability of students to better understand the abstract concepts of mathematics by visualizing them through the physical objects around them in the real world. From the evaluation results comparing the pre-test and post test, it is ascertained that this method of learning acted as a better memory recaller and also built a better emotional connect on mathematics. Hence this direction of learning could also lead to a reduction in the fear for mathematics within students.

## 11. Future work

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The core idea of having a physical activity involving real world objects, to connect the mathematics concept with a memory and using visual representation of the concept with real world scenarios and objects can be applied to other mathematics concepts like statistics, Geometry, Permutations and combinations etc.

Future plan is to develop a full fledged application covering the above topics with lesson plan and activities integrated to apply the concepts in real world through augmented reality and evaluate their learnings.

The goal is to make mathematics a subject to be learned without fear and with emotion.

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