P2 Project Report

Visually Understanding Theory of Relativity

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Prototype
https://jribh.github.io/Spacetime-Jribh/

Introduction

Relativity is as widely known as it is improperly/partially understood amongst the masses. Even amongst circles with sufficient interest in physics, General relativity is often demonstrated with the help of a 2D surface with matter resting on top of it. This representation is simple but erroneous [15], one of the many errors being its inability to show neither time nor the third spatial dimension. Proper visualization of space-time warping is challenging especially on a 2D interface, hence we are stuck with oversimplified explanations. The problem with visualisations pertaining to explain relativity is that they are either oversimplified, or overly complex requiring mathematical knowledge of differential geometry [19]. I aim to reach a middle ground, explaining relativity using visualisations that are conceptually accurate while being understandable by non-physicists. My goal is not to design something new altogether, but to consolidate the ideas already in circulation in scientific and non-scientific arenas, and present them in a way that's easy to understand.

I want the reader to note that relativity is not "just another discovery" within physics. It rather is something that completely turns classical physics upside down. It deals not only with science, but the efforts of a man to overturn centuries of assumed notions. It teaches us how to question everything and not to take anything we have ever been taught for granted, even if it involves re-thinking about the very existence of time and space.

Additionally, efforts to teach relativity to an audience with non-scientific background are often dismissed citing that the topic is an advanced one that doesn't affect a normal person's day to day activities, or rather that they can't be "experienced through experimentation". Although it is true that the most effects of relativity, especially those demonstrated in Special Relativity such as relativistic time dilation, become apparent when one reaches close to atleast one-third the speed of light (something none of us have experienced yet), one should not forget that it is the effects of General Relativity (the earth and the sun curving spacetime around them, for instance) that lead to gravity we all experience every moment. I believe that an effort should be made to make the truth clearly available and accessible to the masses that not only explains what they don't know, but also *what they don't know they don't know*. Experiments substituted with interactive visualisations would offer immersion, while being fun and instructive, providing a highly motivated introduction to the theory of relativity, especially to students [19].

Relativity deals with highly unintuitive and difficult to visualise concepts, and creating true visualisations, especially on a 2D screen, is impossible. All one can attempt are abstractions which best explain the observed phenomenon. In this project I provide science enthusiasts with an online interactive article that visually explains Special and General relativity, focussing specifically on space and time. The article aims to explain the topics in a narrative structure, such that most questions that arise in the reader's mind are explained within. For making the article self explanatory, I go into many subtopics in sufficient depth. Thus, I expect the reader to invest sufficient time in the article to properly grasp the concepts.

The article can be accessed here: https://jribh.github.io/Spacetime-Jribh/ (best viewed on Chrome for Laptop/Desktop).

Aims and Objectives

The main objective of the project is to make a product that would enable the participant (the user) to grab a basic understanding of the concepts of space and time of Special and General Theory of Relativity, with the help of creative and interactive visualisations and interactions. I take ideas and illustrations that aim to explain relativity from multiple sources ranging from books to research papers, simplify them, add interactivity wherever possible for easier understanding, and present them to the audience as a consolidated package.

The product consists of an interactive online article that aims to explain to the participant the following areas of the theory in detail:

1. Understanding time dilation in absence of gravity. Interactions are designed to understand relationship between velocities of objects, time and the speed of light. This section explains the

basics of **Special Relativity**.

2. Understanding the behaviour of spacetime in presence of masses; visually show the spatial and time dimensions and how they interact with each other. Interactions and animations are designed to make people understand how the spacetime fabric curves and curls in presence of mass. This section would cover **General Relativity**.

3. How is gravity caused by time dilation? Visually exploring the origin and occurrence of gravity.

Scope

The aim is to create an instructional package consisting of a web portal that, using an interactive article, would seek to visually explain General and Special relativity to the audience.

Although the product would be aimed for everyone regardless of a formal scientific background, considering the fairly advanced understanding, ability to understand correlations, and imaginative skills required for most of the sub-topics, the product would specifically serve a user-base in the process of finishing or having finished high-school education. Students, avid readers of general level science, teachers and even professionals can find the article of interest, not just for learning, but also for exploring the ideas from a new perspective.

Methodology

I follow an iterative design process, allotting maximum time to ideating and prototyping (see Figure 1). After an initial research phase for solidifying my basics, I keep dwelling into secondary research throughout the process of design. Individual methodologies of each phase are given below.

Secondary Research

1. Topic specific- Researching through books, papers, articles and other resources pertaining to Theory of Relativity to develop my own understanding.

2. Visualisation specific- Research to understand best practices for visualisation.

3. Research to understand which interactive techniques maximise engagement and are most effective for retaining knowledge. For example, comparing sliders/scroll/still images/videos and input boxes.

Design

For ideation phase, I use a combination of techniques like sketching and digital prototyping using tools such as different JavaScript libraries (D3js, Seen.js, three.js and so on), video editing tools, digital animations and so on. For the final product, I use HTML, CSS and JS to make an interactive web article.

Product Testing

I understand that domain knowledge is indispensable [19] when dealing with scientific visualisations. Thus, I ensure fruitful discussions with domain experts in Physics and Relativity at multiple stages of the project.

For evaluating the design, especially the presentation techniques suitable for popular science, I contact subject experts once I have the prototype in place.

For testing the effectiveness of the final product I keep topic specific tests for the target audience, to see how effective my product is in explaining the topic.

Literature Review

Properly understanding Relativity myself was vital in being able to visualise it for others. The literature covered ranges from books explaining the basics (like Relativity by Einstein and Visualising Relativity by Epstein) [4] [6], all the way to research papers from modern physics that use state of the art technologies to visualise relativistic phenomenon.

Since Einstein gave the revolutionary suggestion of space and time not being absolute in 1916, there have been multiple attempts to

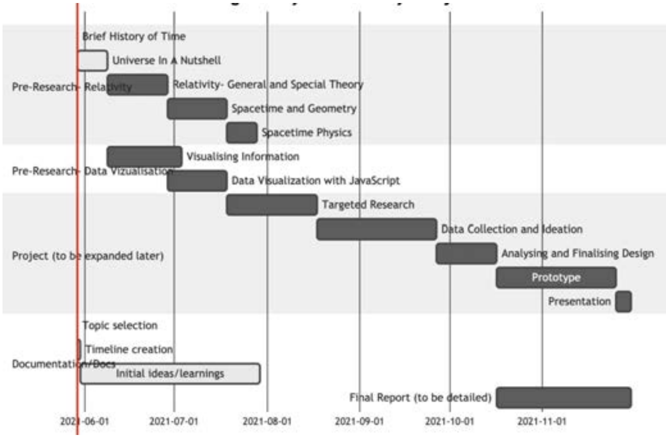

Figure 1: Project Timeline

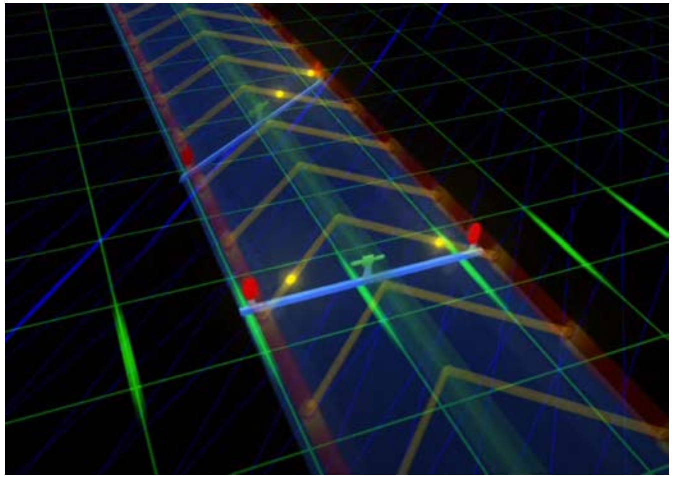

Figure 2: Photon Cart Visualisation

simplify it to be able to make the general public understand the whole hype. As Einstein talked just in numbers and complex mathematical formulae, physicists and artists tried their best to convert that mathematics into conceptual visuals, some of which I would be discussing here.

Special Relativity

One of the first visualisations for special relativity was given by Einstein himself, in which he imagined a moving train on a platform with two observers, one being inside the train and the other stationary on the platform. In this scenario Einstein asked what would happen if the passenger shines a beam of light, since light travels at a constant speed for everyone. This scenario ultimately leads to the fact that time itself is relative for both the observers [4].

Special relativity and the absolution of simultaneity are also visualised using a photon cart, in which a moving cart is imagined to have a photon emitter in the middle and two mirrors at the end (see Figure 2). As the cart moves, it is shown how simultaneous events are no longer simultaneous if one changes the reference frame [18]. Since this visualisation is mostly shown as a static image or a video with zero interactivity, it becomes difficult to understand how speed actually affects simultaneity. I introduce interactivity to this visualisation to make it easily understandable.

Apart from static visualisations, there have been multiple attempts at creating interactive games that show the effects of Special Relativity in real time. "A Slower Speed of Light" is a game developed by MIT Game Lab, that imagines a world where light moves at a much slower speed (see Figure 3). It shows the effects of special relativity including time dilation, relativistic abberation and doppler shift via a first person camera which the player can control [14]. Another attempt has been a carrom billiards game that shows real time interactions between objects moving at relativistic speeds [3].

Speaking of visualisations used to demonstrate Special Relativity in actual physics, one of the most common ones are worldline diagrams [8]. These diagrams show the movement of objects through

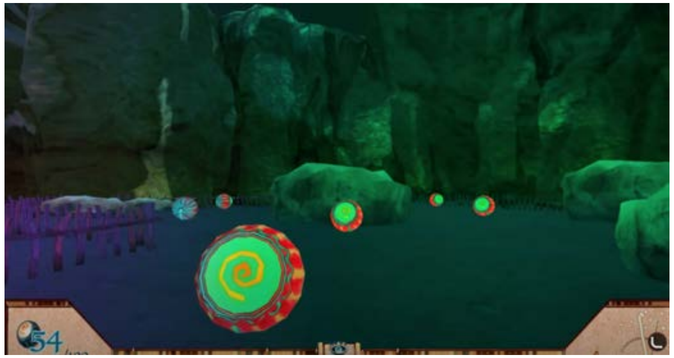

Figure 3: A Slower Speed of Light

space and time, while allowing the user to compute exact time dilation and length contraction.

General Relativity

General relativity majorly comprises of effects of matter on 4 dimensional spacetime. Visualising or even conceiving this in its exactness is, as I have iterated before, impossible. There have been, however, multiple attempts to abstract some of the concepts and present them in intuitive ways.

One of the most famous and widely used visualisations is that of a stretched fabric, on which matter, or more specifically, planets are placed as spheres (see Figure 4). Due to the 'weight' (since the experimenters place the spheres on the sheet, the sheet bends down due to the weight of the spheres) of the planets, the fabric bends down, thus pulling other planets towards it. This visualisation is easy to demonstrate and shows the attractive nature of gravity. It, however, is flawed in the sense that it seeks to explain gravity using gravity. It doesn't explain the source of gravity that is pulling these planets down on the fabric [6]. Apart from this, it also fails to include the dilation of time caused by the presence of matter.

Spacetime curvature has also been visualised using what are called

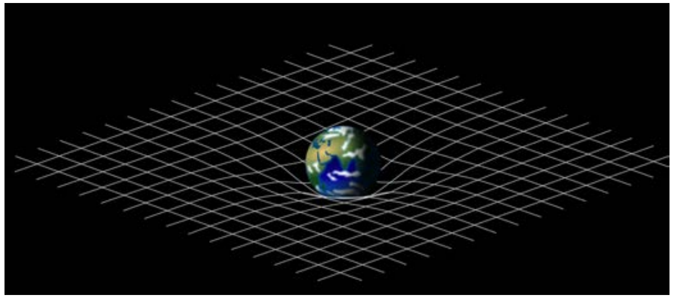

Figure 4: Stretched Fabric Visualisation

embedding diagrams. These are basically rolled up versions of spacetime diagrams that show one spatial dimension and the time dimension. These show the dilation of time caused by matter, and explain the cause of gravity being the gradient in time [10]. Although it is restricted to just one spatial dimension, it is more scientifically accurate than the stretched fabric visualisation, helping gain intuitive insights into the gravitational field rendered by curved spacetime [9]. Unfortunately, embedding diagrams haven't yet made it to popular science's representation of gravity, which is a shame considering their higher scientific accuracy. They mostly reside in physics research papers which are not easily accessible to the general reader. I make use of these embedding diagrams, simplify them to a certain level, and add interactivity to help the reader understand the effects of mass on spacetime in real time.

Advancements in computer graphics and animations have allowed us to visualise spacetime curvatures to a degree of remarkable accuracy. One way of showing all four dimensions of spacetime getting distorted by mass is showing a large celestial body (say, the earth) in a cubical grid. The grid constantly moves inside the earth, which shows the spatial distortion caused by the mass. The dilation in time is shown by dynamically moving clocks, with the clocks closer to the earth moving slower than the ones further apart [5] (see Figure 5).

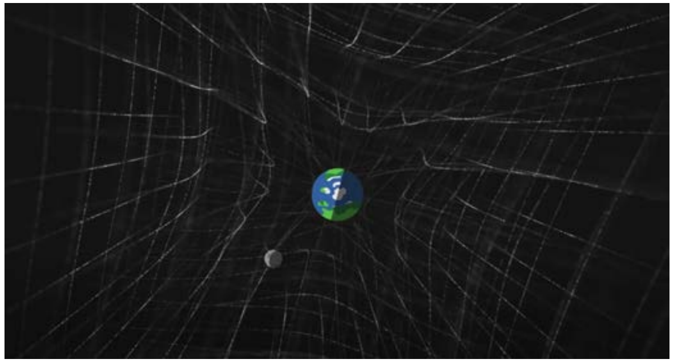

Figure 5: Modern Visualisation of General Relativity

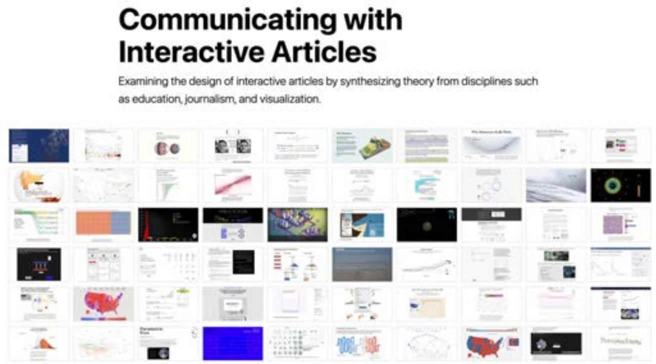

Figure 6: Communicating with interactive articles

Data Visualisation
Speaking strictly of best practices to follow when visualising complex phenomenon via a limited medium, one has to ensure that only the most relevant dimensions are presented. This helps in not only simplifying messy graphics, it also brings forward the most important characteristics of the actual data and reduces cognitive load [19]. I use this knowledge extensively, especially while describing the 4 dimensional spacetime of General Relativity, by extracting just two or three most relevant dimensions that easily demonstrate the phenomenon I seek to explain. Apart from this, I try my best to preserve data density and not oversimplify concepts. The data shouldn't be dumbed down to cater to the less knowing- rather the design should enable them to understand it with applied effort [17].

The technique of presenting similar datasets as small multiples has been emphasized, as that enables the viewer to easily make comparisons. Speaking of enabling comparisons, numbers and colour quantifiers (or, choropleth diagrams) serve as some of the best methods to show gradually changing data [17]. I make use of these when visualising the smooth gradients in time.

Visual depictions of phenomena like relativity can be scientifically correct, but not always sound. For instance, a worldline diagram visualisation of special relativity showing reduced and only relevant dimensions might not clearly convey the concepts to the untrained mind. To combat this, I try to keep a balance between scientific accuracy and soundness by using a mix of exocentric visualisations with simple mathematics and medium complexity, and egocentric visualisations with mimicry based on well known scenes [19].

Lastly, I understand that the aesthetic-usability effect [13] is important especially when catering to a larger audience. Aesthetic and attractive visualisations, while being appealing, also promote creative thinking and problem solving [19].

Interactive Online Articles
Online articles are a great way of reaching a large audience. Combined with the power of modern web technologies, interactive online articles are a great way of teaching and visually showing complex topics. Communicating With Interactive Articles [1] by Distill does an excellent job of curating such articles (see Figure 6).

In this section, I look at some of the most well designed articles that seek to explain difficult topics in a visual and interactive manner. A Visual Introduction to Machine Learning [16] uses minimal text and scroll events to make interactive yet unobtrusive and undemanding visualisations to explain a machine learning model to distinguish homes in New York from homes in San Francisco. Why Momentum Really Works [2] explains the mathematics of momentum using clickable graphs and charts.

All of these articles use a combination of different JavaScript libraries to achieve dynamic and interactive visualisations. For my product I use libraries like D3.js, three.js, P5.js and anime.js.

Topic Structuring and Conceptual Visualizations

In this section, I discuss each of the subtopic in detail, along with how I executed the working prototype for explaining each subtopic.

We start with Special Relativity, starting from introduction to the basics such as reference frames and the speed of light, moving on to time dilation. Almost all the visualisations in this section attempt to show the points of view of different reference frames, something crucial to Special Relativity. After establishing the basics in controlled environments, we then introduce acceleration and move on to General Relativity, where we talk about and visualise curved spacetime and gravity.

Topic structuring is done in a way to maximise engagement and learning, such that the questions raised by a certain section are answered in the next one, thus maintaining a flow. The topics are as follows:

1. Special Relativity:
   - (a) The speed of light (c) is constant. Why?
   - (b) What are reference frames?
   - (c) Restricted Principle of Relativity
   - (d) How can the speed of light be constant for variably moving reference frames? By non-absolute time
   - (e) What is simultaneous? - Photon plank moving in space
   - (f) The problem - Photon plank moving in space and time shows photons travelling at different speeds
   - (g) Space and time are one
   - (h) Everything travels at c through spacetime
   - (i) Photon tube- The cosmic speedometer
   - (j) Spacetime plot (speedometer) to see the effect of relative speed on length and time experienced
   - (k) Lorentz Transformations showing time dilation and length contraction
   - (l) Worldlines
2. General Relativity and Curved Spacetime:
   - (a) Accelerating reference frames- Equating gravitational and inertial mass
   - (b) Rotating disc- Accelerating reference frames aren't Euclidean
   - (c) Straight Lines and Multiple Dimensions
   - (d) Mass curves Spacetime- Since accelerating reference frames are equal to gravitational fields
   - (e) Object worldlines in curved spacetime- The "force" of gravity
   - (f) Embedding diagrams- rolled up versions of curved spacetime
   - (g) Embedding diagrams of different masses
   - (h) Gravity in Embedding Diagrams
   - (i) Light in embedding diagrams- Mass increases space along with dilating time
   - (j) Visualisation showing time and space curvature together
   - (k) Light curves in gravitational fields
   - (l) Visualising all 4 dimensions of spacetime

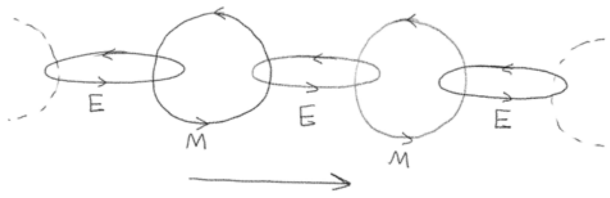

Figure 7: Light Propagation in Vacuum: Initial Sketch

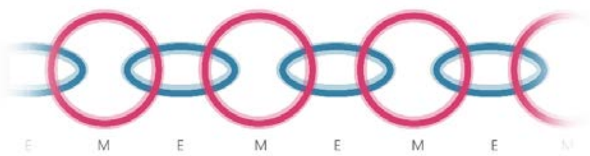

Figure 8: Light Propagation in Vacuum

Special Relativity

The Speed of Light is Constant- Why?
The speed of light, or more specifically the speed of causality, at which massless particles like light happen to travel at, is constant. Why?

Light is a wave formed by propagation of alternating magnetic and electric fields. It must move to exist. If its speed were variable, there would be a scenario when you could perceive it to be stationary. That would be impossible since the alternating fields must constantly propagate, and should never be stationary. Thus, c is constant. This fact is important and forms the backbone of the Theory of Relativity (see Figure 7 for the initial sketch).

The final prototype shows an animation of a chain of alternating electric and magnetic fields, as can be seen in Figure 8.

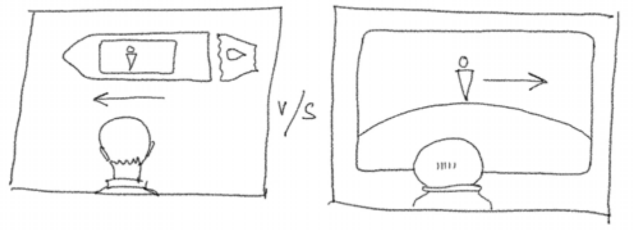

Figure 9: Reference Frames: Initial Sketch

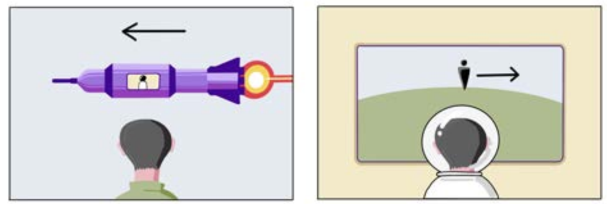

Figure 10: Reference Frames

What are reference frames?
In this section I introduce to the reader reference frames, ie. different points of view through which events can be seen. I compare the reference frames of two friends, one who is stationary and the other who is in a moving rocket (only uniform linear motion).

I show how no reference frame is actually at absolute rest, as the friend in the rocket can as easily perceive the person outside to be moving (see Figure 9 for the initial sketch, and Figure 10 for the final illustration).

Restricted Principle of Relativity
I introduce the principle of relativity which states that no reference frame is special (as we saw in the visualisation above), and that all

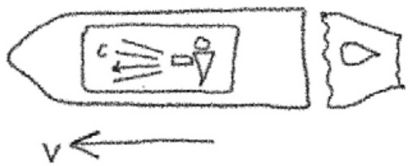

Figure 11: Light in Moving Reference Frame: Initial Sketch

Figure 12: Light in Moving Reference Frame

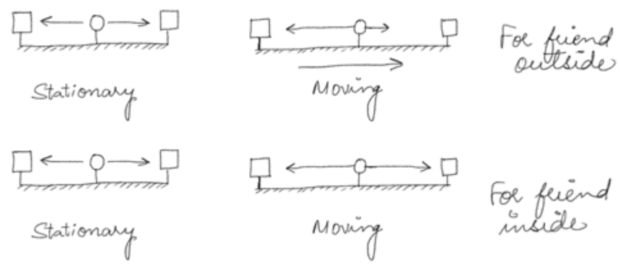

Figure 13: What is Simultaneous?: Initial Sketch

reference frames must have the same laws of physics. It's restricted because at the moment we are only talking about uniformly and linearly moving reference frames.

How can the speed of light be constant in different reference frames?
We just saw that no observer should measure light moving at a speed other than c. But, what if the friend in the spaceship lights up a torch? Will the speed of light add up with the spaceship's speed for the outside observer? (see Figure 11 for the initial sketch, and Figure 12 for the final illustration)

The answer is it won't. The speed has to remain constant, and for that, something has to give way. That something is time.

What is simultaneous?
Here I show that simultaneous events from one reference frame may not be simultaneous from another reference frame. I show this by placing a photon emitter (or, simply, a torch) in the middle, and two mirrors at the ends of a plank. I then put this plank inside the spaceship we saw earlier (see Figure 13 for the initial sketch, and Figure 14 for the final illustration).

As can be seen from the reference frame of the friend outside, light takes longer to reach the right mirror the faster the spaceship is moving. This is because, after the light is emitted, the mirror on the right has moved further away, and light has to travel a greater distance. On the other hand, the friend inside the spaceship sees the light hitting the two mirrors simultaneously, as the plank is at rest compared to him, and only the world outside his window is moving.

The plank also moves through time
Since all objects are moving through time, even the stationary ones, I introduce time moving upwards in the Y axis, behind the plank. Now, when the user changes the speed of the plank (or the spaceship) through space, The plank moves through space and time both, thus moving diagonally.

Here, I use small multiples to show the expected v/s the actual movement of the plank (see Figure 15 for the initial sketch). The expected movement, shown in the sketch in the top right figure, keeps the plank size constant and the events simultaneous. But it doesn't keep the speed of light constant.

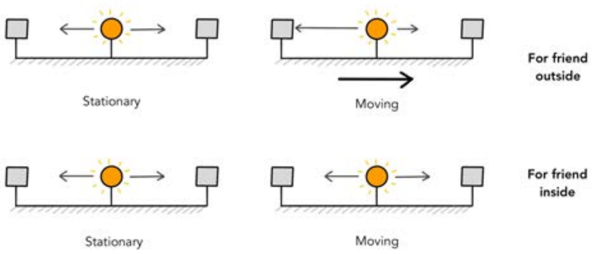

Figure 14: What is Simultaneous?

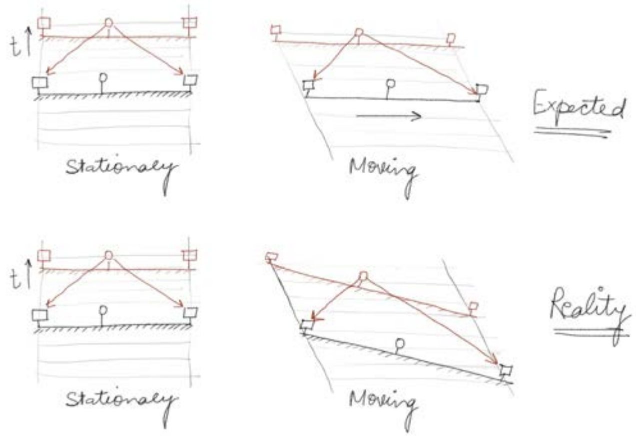

Figure 15: Photon Plank Through Time: Initial Sketch

To preserve the constancy of light speed, I show what happens in actuality, in the visualisation on bottom right. Events are no longer simultaneous and length is no longer constant.

Figure 16 shows a screenshot of the final working prototype. The speed slider increases interactivity and shows the distortion of length and time in relation to the plank speed.

The red plank is as it was in the past, when the light was emitted. The black is as it is not, when the light has reached the mirrors.

It should be noted that all this is as observed by the friend outside. The friend in the spaceship still perceives the plank to be still from his own reference frame, which is the one on the left.

Space and time are one
As seen from the previous visualisation, desynchronization in time is inevitably accompanied by length foreshortening. This leads us to the revolutionary suggestion by Einstein- Space and Time are not independent, but one. You can't change one without affecting the other. Whenever you move, it's never exclusively in space or time. It is ALWAYS in SPACETIME (see Figure 17 for the initial sketch and Figure 18 for the final illustration).

Although the previous visualisation attempts to mix space and time to show spacetime together, it should be noted that a perfect visualisation of 4D spacetime is IMPOSSIBLE to draw or conceive. We can, however, extract one or two spatial dimensions and show them with time. With proper abstraction, this would allow us to explain a lot of concepts regarding the behaviour of spacetime.

Everything travels at 'c' through Spacetime
Here is where the “assumption” comes in, which is that everything travels through spacetime at the speed of light c, nothing less and nothing more. Stars, planets, atoms, trees, me and you, all are ALWAYS hurling through spacetime at the maximum possible speed. Even stationary objects are moving through time, as we remember that space and time are not independent. If an object moves through space, it has to divert some of its speed through time to move through space. Thus, it moves through less time.

I visualise this using a cosmic speedometer, which is a graph showing one spatial dimension and the time dimension (abstractly shown for now) (see Figure 19 for the initial sketch and Figure 20 for the

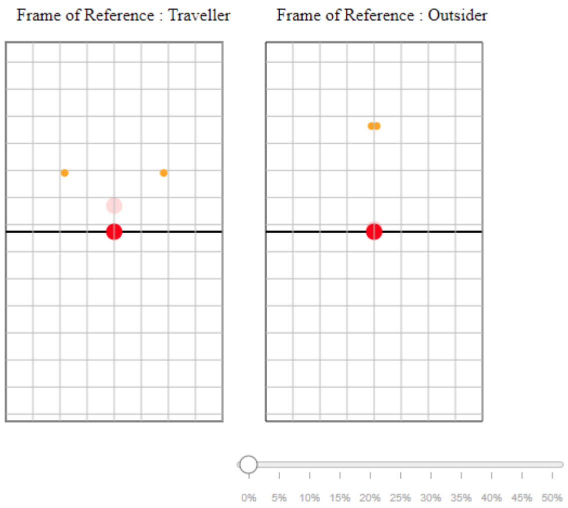

Figure 16: Photon Plank Through Time

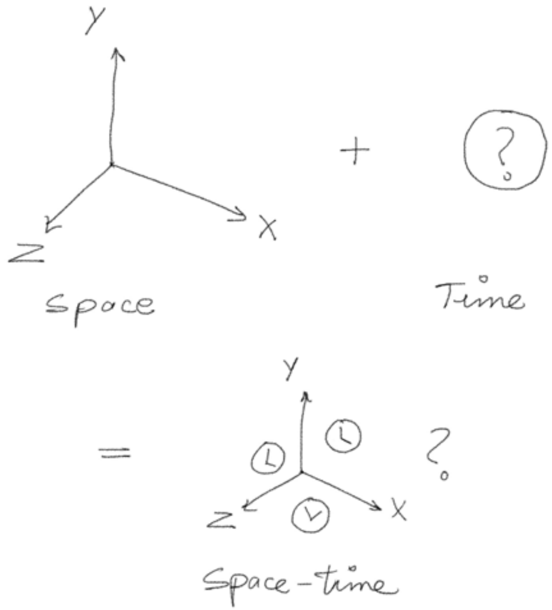

Figure 17: Space and Time Are One: Initial Sketch

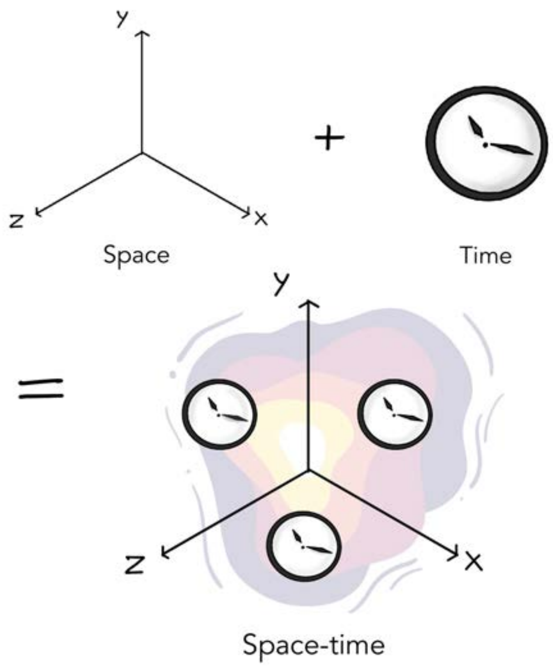

Figure 18: Space and Time Are One

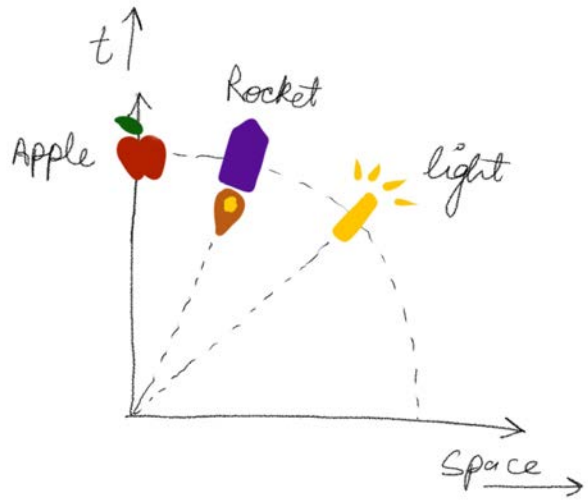

Figure 19: Everything travels at 'c': Initial Sketch

final illustration).

Photon tube- The cosmic speedometer
Why do fast moving objects move through less time? I visualise this using a photon tube (basically, a tube with a light torch) (see Figure 21). We once again place this on the spaceship of one of our two friends.

The first visualisation shows a tube which is 1 light second long (ie. light takes 1 second to reach from one end to another, if the tube is stationary). I then provide an option for the user to move the tube with a constant velocity, all the way from 0 to c. As the tube moves faster and faster, one sees that the light beam is able to cover a lesser distance, as it moves with a constant velocity. On the time axis, we see that our tube has moved through time that is less than one second, thus proving our point that objects moving faster in

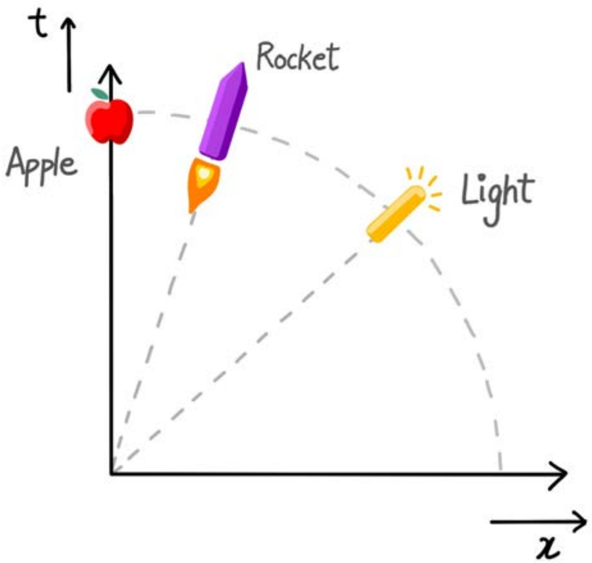

Figure 20: Everything travels at 'c'

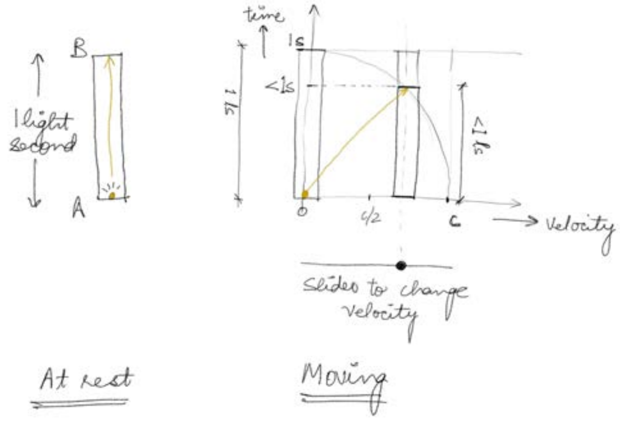

Figure 21: Photon Tube: Initial Sketch

space move through less time. Along with that, the apparent length of the tube also appears shorter the faster it is moving (the fore-shortening effect). Note that all this is from the reference frame of the outsider. The friend on the spaceship still views it to be stationary, as seen on the tube on the left.

One can also see that, as the speed of the tube approaches the speed of light, it stops moving through time, as the light can never reach the other point. Also, at the speed of light, the size of the tube becomes zero, and both the end points coincide. A screenshot of the final working prototype can be seen in Figure 22.

Spacetime plot (speedometer) to see the effect of relative speed on length and time experienced
Consider the spaceship one of our friends is in to be moving at a uniform velocity. The velocity is controllable by the reader for better immersion and understanding (see Figure 23 for the initial sketch and Figure 24 for the working prototype).

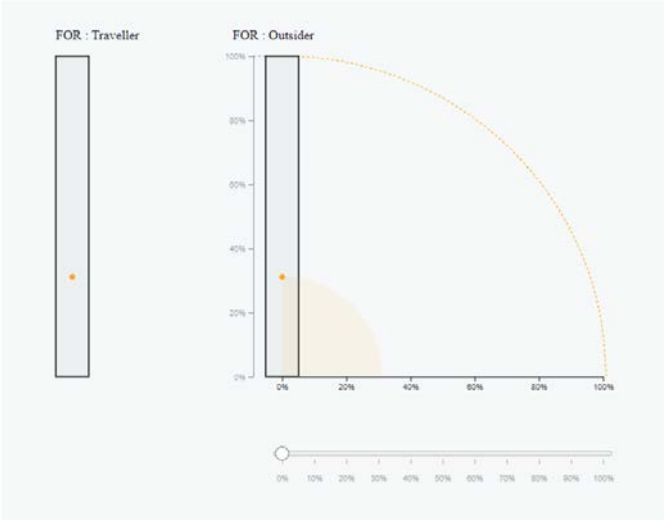

Figure 22: Photon Tube

The reader would notice how the length contracts and the space-ship moves through lesser time, the faster it is moving. It should be remembered that, as everywhere in the realm of special relativity, we are *only considering linear unaccelerated motion*.

Lorentz Transformations using the Cosmic Speedometer
Lorentz equations enable us to calculate how much time dilation and length contraction occurs at a given velocity. I visually show this using the cosmic speedometer we just made (see Figure 25 for the initial sketch).

The velocity slider can be adjusted from 0 to c, enabling the reader to see exactly how much time is dilated and length is contracted at a particular speed.

The sketch shown in Figure 26 shows the effect of time dilation and length contraction when the spaceship moves at a velocity v, as seen from the reference frames of the two friends, one inside and

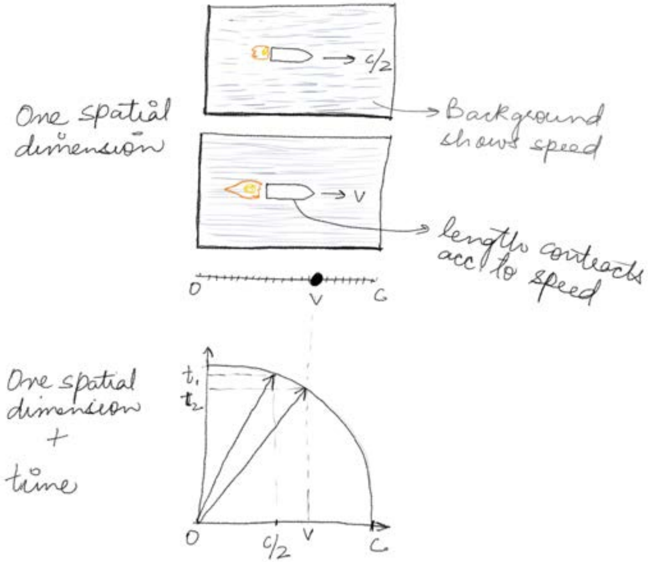

Figure 23: Visualisation to compare spacetime diagrams of two spaceships

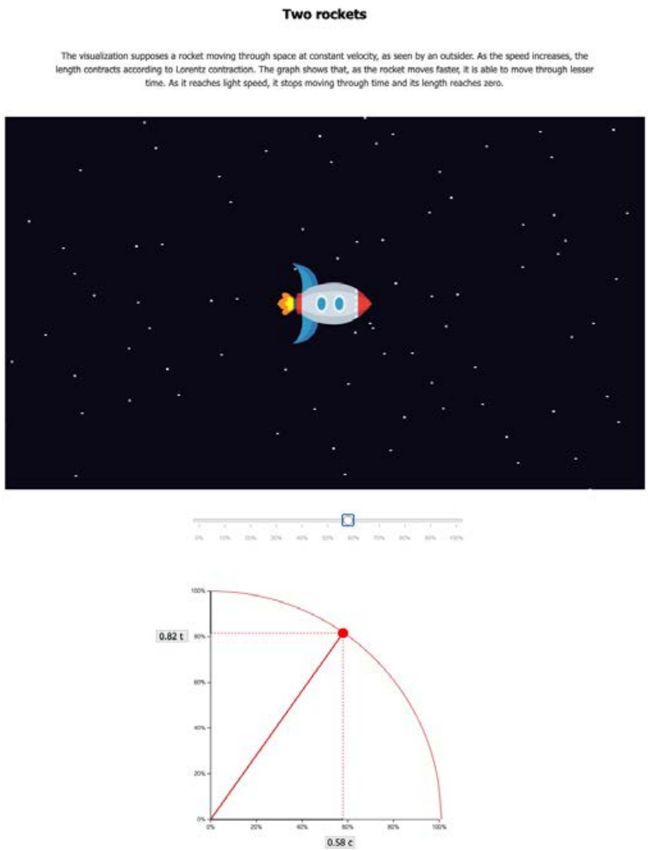

Figure 24: JavaScript Visualisation for length contraction and time dilation

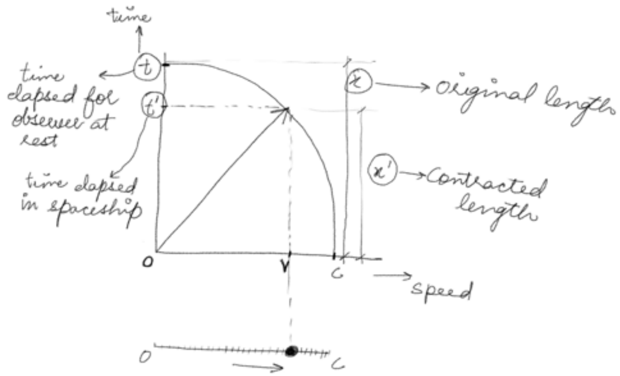

Figure 25: Cosmic Speedometer

one outside the spaceship.

This reinforces the fact we talked about earlier- no reference frame is special. Just as the friend outside sees the time slowing down on the spaceship and the length getting contracted, the traveller observes the same for the world outside, for he has no reason to believe that he isn't at rest and the world outside isn't moving backwards.

Worldlines and light cones

A cosmic speedometer is easy to understand, but physicists prefer spacetime diagrams where velocity is represented by slope. These diagrams show a single spatial dimension on one axis, and the time dimension on the other.

As all objects are constantly moving through spacetime at a velocity c (which we just discussed), they follow a path through this diagram, depending on their speed through space. This path is called a worldline. The worldline of a stationary object is a vertical line as it moves just through time. The spatial and temporal dimensions are chosen such that the world line of light is a 45 degree inclined line.

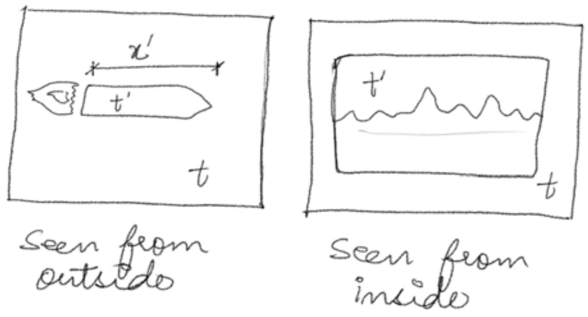

Figure 26: Lorentz contractions from different reference frames

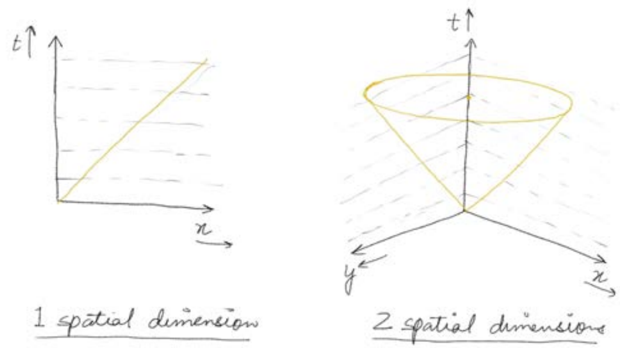

Figure 27: Light Cone: Initial Sketch

If we include the Z axis to show another spatial dimension, we can extrude this 45 degree line to form a cone. This is called a light cone (see Figure 27 for the initial sketch and Figure 28 for the final illustration). All objects, moving or stationary, must have their worldlines inside this cone, as they can never approach or exceed the speed of light.

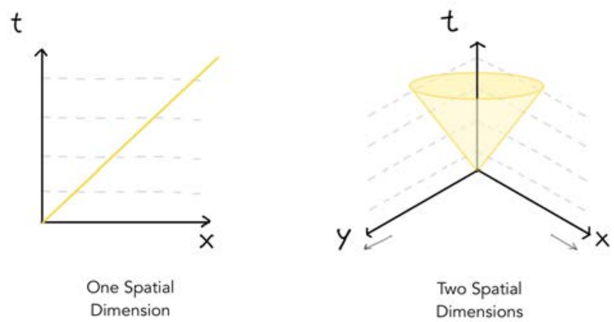

Figure 28: Light Cone

General Relativity and Curved Spacetime

Accelerating Reference Frames

Up until now, our focus has been linearly moving and non accelerated reference frames. But what happens if the reference frames are accelerating or rotating with reference to each other? For instance, what would happen if our friend in the rocket ship decides to accelerate or turn his ship, while his friend outside doesn't move? How would time and space behave for both of them?

Einstein says that the force one feels inside an accelerated spaceship is the same as would be felt if the spaceship was stationary, with a variable gravitational field acting on it. In other words, accelerating reference frames can be thought of as stationary if one believes that an external gravitational field is acting on the frame. This allows us to apply the principle of relativity, which we just discussed,

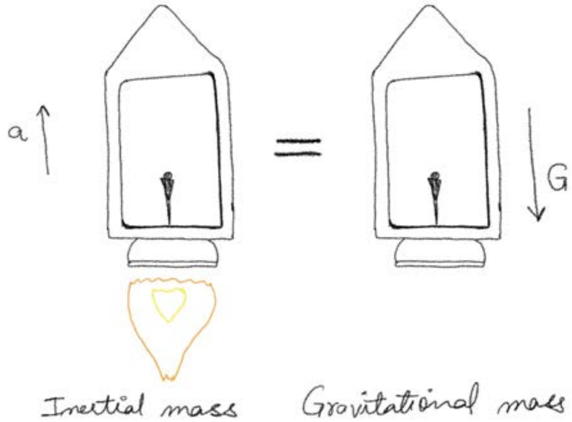

Figure 29: Accelerating Reference Frame: Initial Sketch

in a more GENERAL sense, ie. all laws of physics hold true for such a reference frame, including the constancy of velocity of light.

I show this by visually equating an accelerating rocket ship with a rocket ship in a gravitational field. This allows the user to conclude with the following important insight, inertial mass = gravitational mass (see Figure 29 for the initial sketch and Figure 30 for the final illustration).

A Rotating Disc
Let's look at further implications of accelerating reference frames on spacetime. Let's consider a huge disc that is rotating around its center. All points on the surface of the disc are in non-uniform motion, apart from the center, which is stationary.

I show the visualisation of a disc rotating, with an interactive slider allowing the user to control the speed of rotation. The speed can be such that the outermost edge of the disc can almost but not completely reach the speed of light (see Figure 31 for the initial sketch).

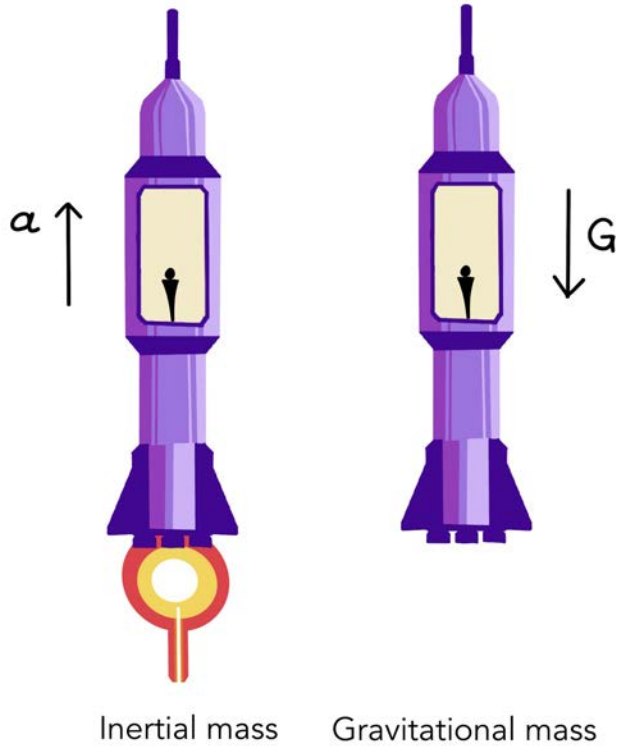

Figure 30: Accelerating Reference Frame

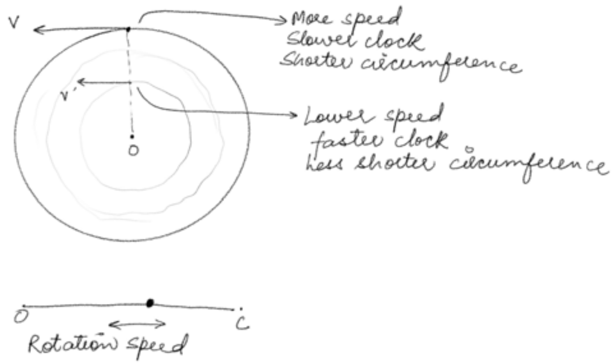

Figure 31: A Rotating Disc: Initial Sketch

Figure 32 shows the final working prototype of the disc.

Now, on a rotating disc, the outermost point is moving faster with respect to the points closer to the center, with the center always being stationary. Hence, according to our knowledge from Special theory of relativity, the points moving faster should experience a slower clock. Also, the circumference of the disc should be measured differently by two observers who are at different distances from the center, due to relativistic length foreshortening, although the radius would be the same.

This clashes with our knowledge of classical geometry, also called Euclidean geometry, where a circle of a certain radius has to have a certain circumference.

This leads us to conclude that space and time don't behave as expected when reference frames are accelerated. A straight line isn't always straight, and triangles aren't always planar. Think of it as a napkin. A line drawn on a flat napkin appears to curve if the napkin is wrinkled.

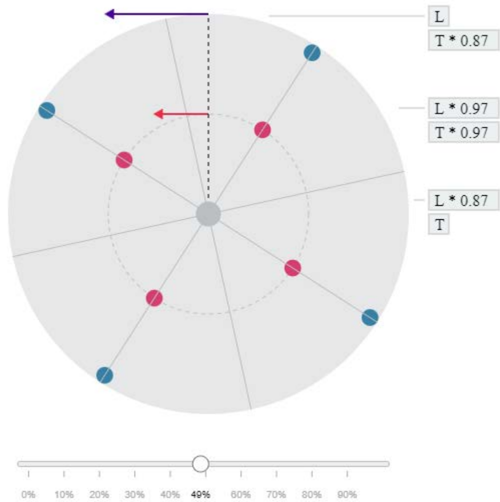

Figure 32: A Rotating Disc

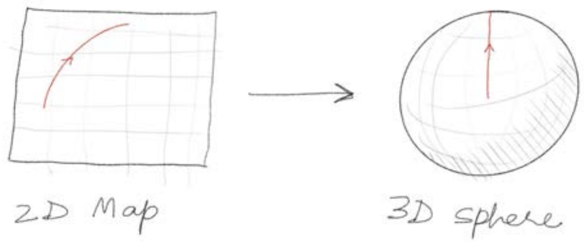

Figure 33: Straight lines in different dimensions: Initial Sketch

This revolutionary suggestion by Einstein, that spacetime isn't Euclidean, led to major implications which we will now discuss.

Straight Lines and Multiple Dimensions

What is a straight line? Consider that we draw a straight line on a flat piece of paper (that's two dimensional). Now, if we roll the paper into three dimensions, the line isn't straight anymore, rather curved. Straight lines can become curved and vice versa when the number of dimensions change. Figure 34 shows how a straight line drawn on a three dimensional sphere appears curved when we flatten out the sphere onto two dimensions. Similarly, a line that's straight in four dimensional spacetime might not be straight anymore when seen in three dimensions. See Figure 33 for the initial sketch.

This knowledge marks the basis of the next section.

Mass Curves Spacetime

We know that accelerating bodies curve spacetime. We also know that acceleration can be substituted with gravitational field, since gravitational mass is equal to inertial mass. Since gravitational fields are caused by the presence of mass, we conclude that mass causes spacetime to curve. This curvature of spacetime in turn dictates the movement of mass in it. Thus, whenever a mass is present, spacetime cannot be Euclidean, but Gaussian (curved).

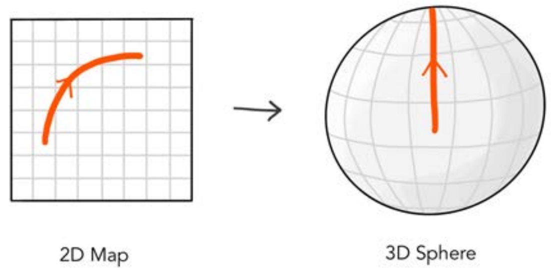

Figure 34: Straight lines in different dimensions

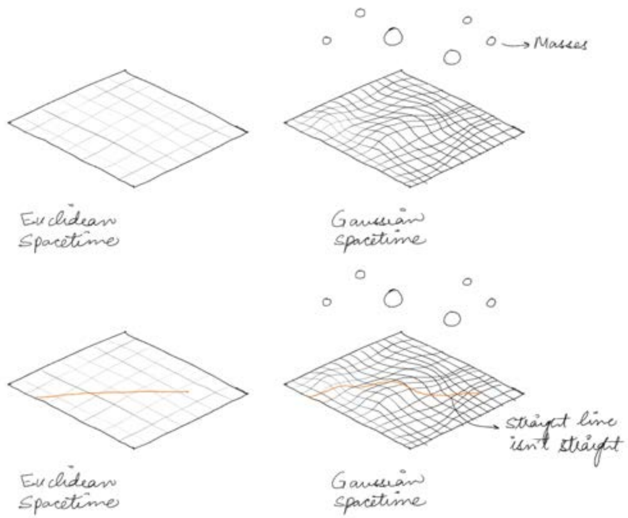

Figure 35: Mass curves spacetime: Initial Sketch

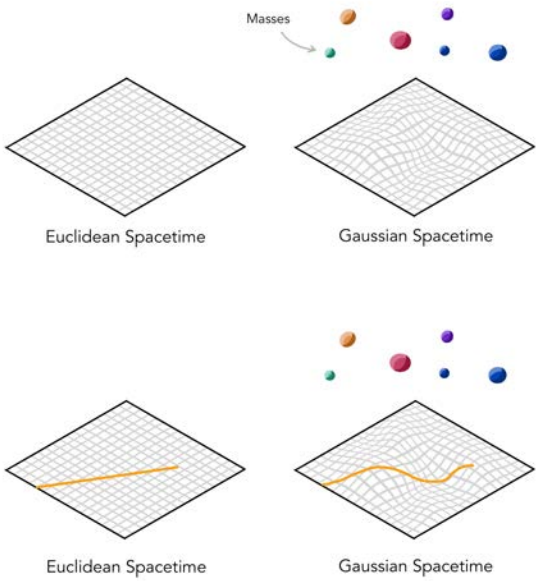

Figure 36: Mass curves spacetime

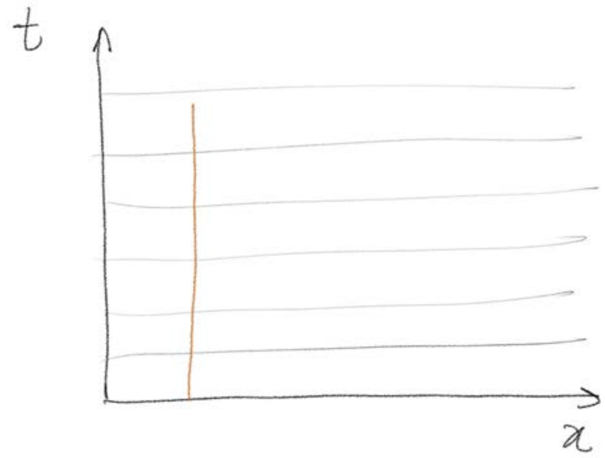

Figure 37: Worldline without gravity

I show this abstractly, with the help of a two dimensional sheet (see Figure 35 for the initial sketch and Figure 36 for the final illustration). I understand that the sheet is not truly representative of how mass would actually affect spacetime, and is used just to intuitively demonstrate how mass causes spacetime wrinkles. It's clear how a line that's straight in Euclidean spacetime isn't straight anymore in Gaussian spacetime.

Worldlines in curved spacetime - Gravity
Let's consider the familiar 2D diagram with space on the X axis and time on the Y axis. Without the presence of any mass, spacetime is Euclidean, behaving like it used to when we were understanding Special Relativity.

Let's consider a stationary object, say, an apple. It would have a worldline (the path it traces on the graph) which is vertical, as it is currently just moving through time (see Figure 37).

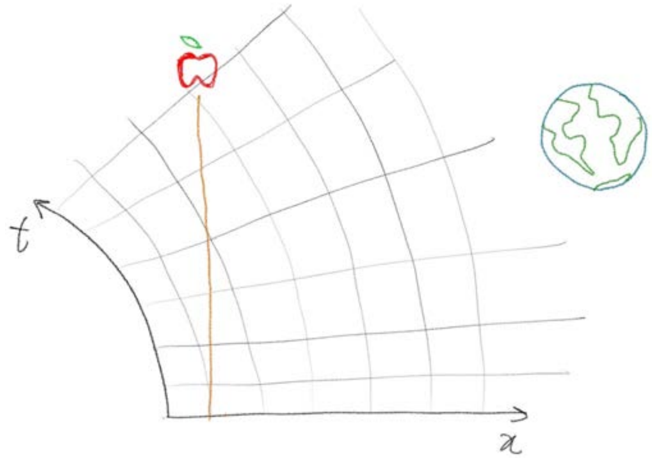

Figure 38: Worldline with gravity

Now, let's place a massive object, say, the earth, towards the right. As we saw, mass causes spacetime to curve. More specifically, the more the gravitational field, the slower time passes. This leads to our spacetime diagram getting distorted, with time passing more slowly to the right [12] (see Figure 38).

Now, if we make a worldline of an apple, it will still be straight, as it was initially at rest and just moving through time. However we see that, due to spacetime now being curved, it has inevitably also moved along space (the X-coordinate), towards the right (towards the earth). This clearly shows the effect of gravity- Earth curving spacetime such that objects initially at rest start falling towards it. See Figure 39 for the final illustration.

Note that, for the sake of understandability, our spacetime diagram only shows one spatial dimension, ie. the distance from the earth.

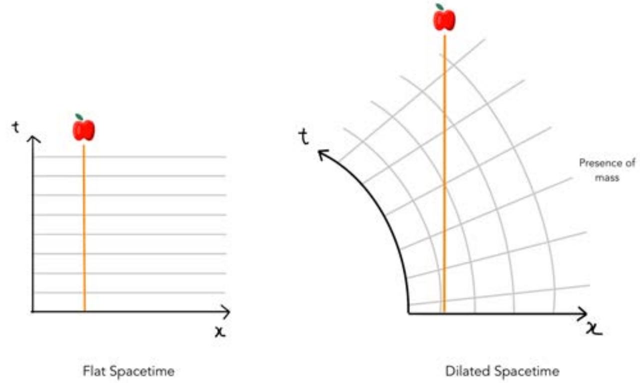

Figure 39: Worldline without and with gravity

Embedding Diagrams- Rolling Up Curved Spacetime

Curved spacetime on a flat surface can be difficult to visualise, even if we restrict ourselves to just one spatial dimension. In this section, I introduce embedding diagrams, which are simply rolled up versions of the 2D curved spacetime diagrams we saw in the preceding section (see Figure 40).

Embedding diagrams help visualise the curvature in spacetime better, are more intuitive, and also explain gravity better (see Figure 45). The visualisation explains how we can get embedding diagrams just by rolling up 2D projections of curved spacetime diagrams. Please note that, although in 3D, embedding diagrams still only show one time dimension and one spatial dimension.

When one literally rolls a flat piece of paper, it results in a spiral fold rather than a continuous cylinder. The same is applicable here. An object moving through time would not end up at the same point in the past after every revolution, but at a different point in the future. We can however, for the sake of simplification, ignore this fold for our purposes, as it would serve to distract the actual purpose of the diagram. The reader, however, has to keep this important fact in

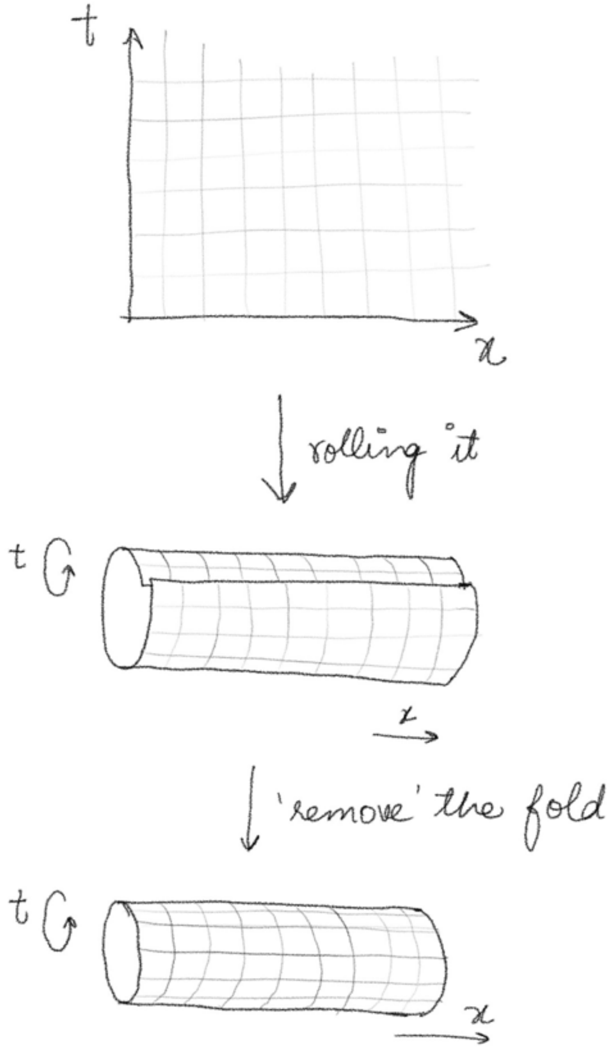

Figure 40: Embedding Diagram for Euclidean spacetime

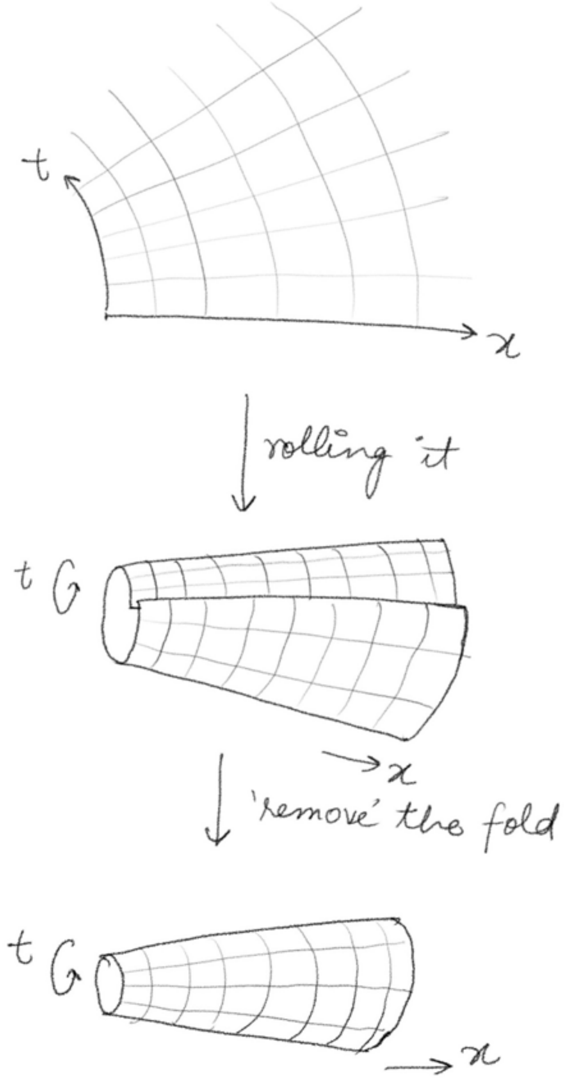

Figure 41: Embedding Diagram for curved spacetime

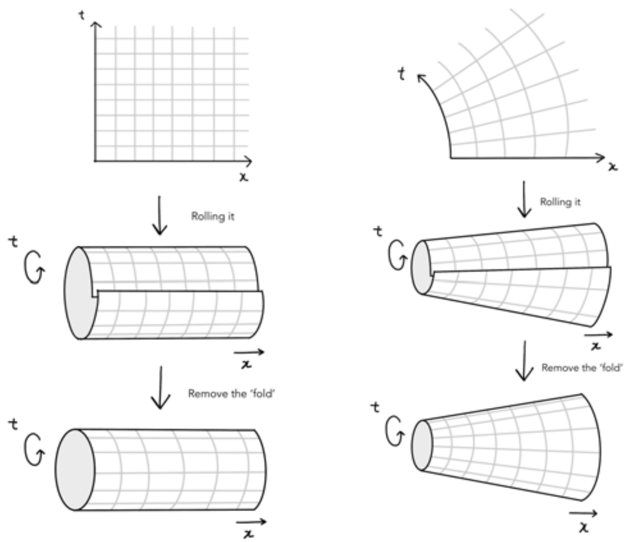

Figure 42: Embedding Diagram for curved spacetime

mind. (see Figure 42 for the final illustration)

Embedding diagrams of different masses
We know that mass causes a gravitational field. The stronger the field is, the slower time runs, and greater the curvature of spacetime is. How does that visually affect our embedding diagrams?

We can categorise the spacetime around a mass (like earth) to be flat and curved, depending on the strength of the gravitational field (though it will be truly flat only at infinity. Mass always bends spacetime around it no matter how far it is). This leads us to a visualisation shown here (see Figure 43). Now, we know that the spacetime curvature is continuous, as the gravitational field increases with strength gradually, as one approaches earth. Thus, we smooth out the edges of our diagram. This type of embedding diagram is called a Dual Embedding [11].

To make this visualisation interactive, I use a huge spherical body of variable mass and radius, with initial values equal to those of our sun. The reader can use a slider to change the mass and radius, and see the resulting effect on the spacetime curvature.

The dotted circle around the object is what's called the Schwarzschild Radius, and it depends on its mass. When an object's radius becomes smaller than its Schwarzschild Radius, it turns into black hole (see Figure 44). This visualisation takes care of that, and attempts to show the black hole embedding diagram as well, as it might appear according to the dual scheme.

Gravity in Embedding Diagrams
Now, let's imagine an apple is kept, initially stationary, at the leftmost point. As it is constantly passing through time, its worldline forms loops around our embedding diagram. Now, due to the curvature of spacetime, the worldline starts drifting towards the right ie. towards the earth. Thus, the apple starts "falling" towards the earth due to the "force" of gravity. To allow the apple to have unrestricted movement, we imagine a vertical hole through the spherical mass, passing through the center. (see Figure 45 for the sketch)

Embedding Diagram Showing Gravity

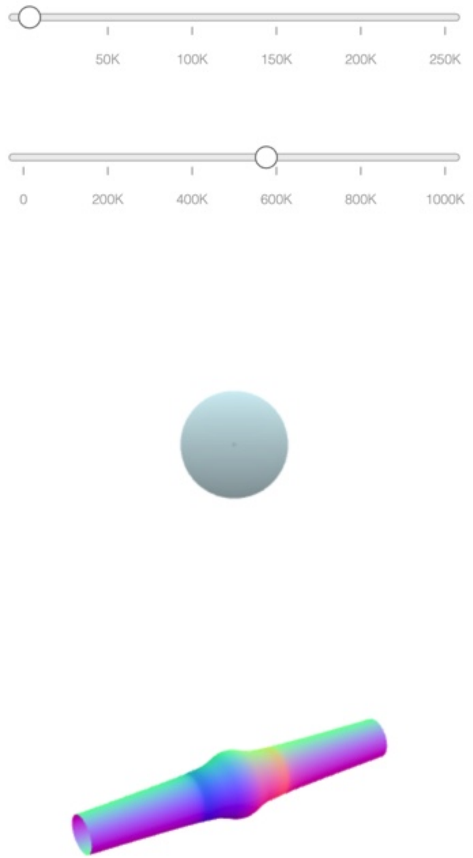

Figure 43: Embedding Diagram of a mass

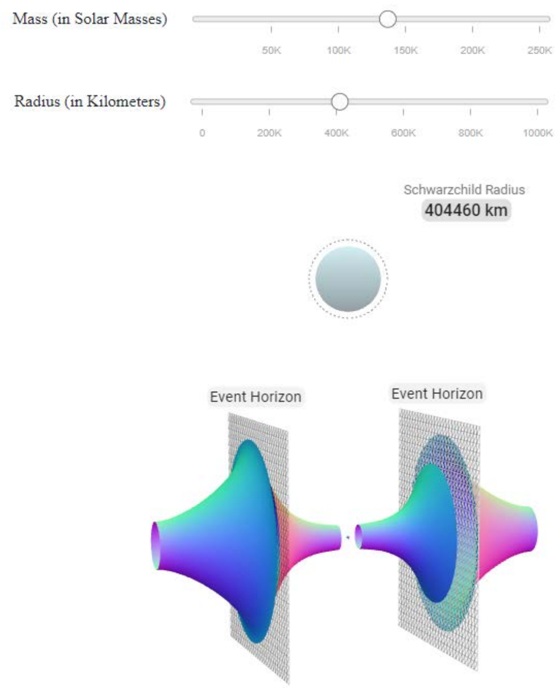

Figure 44: Embedding Diagram of a Black Hole

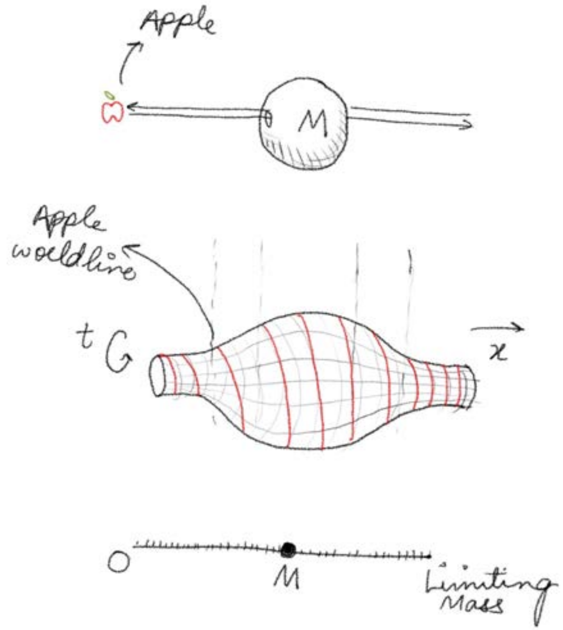

Figure 45: Embedding Diagram showing gravity: Initial Sketch

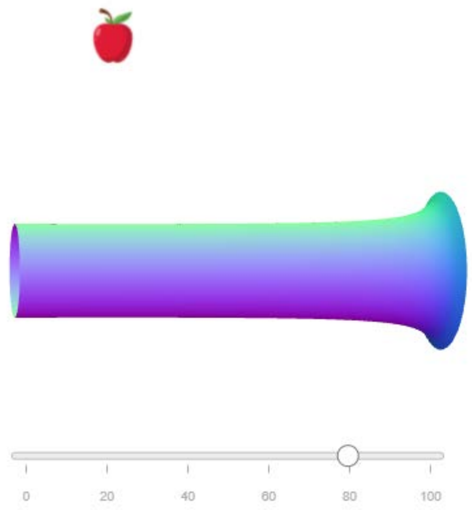

Figure 46: Embedding Diagram showing gravity

Figure 46 shows the working prototype showing gravity in embedding diagrams. As the user moves the slider to the right, time gets more and more dilated, which changes the shape of the embedding. This leads to a gravitational pull on the apple towards the right, which then 'falls'.

Figure 47 shows the geodesic of an apple as it falls towards the right in an embedding diagram. We see that the geodesic curve depends on the amount of dilation that happens on the embedding diagram.

It should be noted that different types of embedding diagrams can

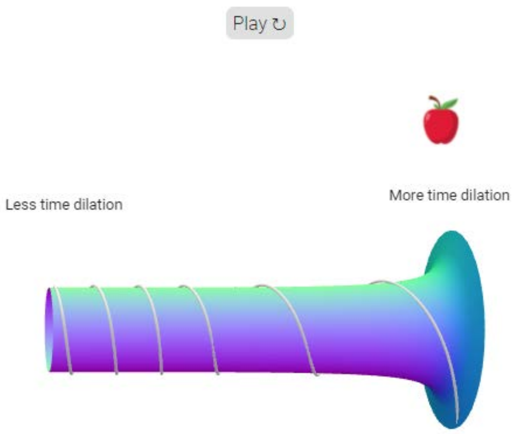

Figure 47: Geodesics in Embedding Diagrams

be made depending on the calculations. The central time dilation can be shown as a bulge (called dual metric) or as a crunch (called absolute metric). Both metrics have their own advantages and disadvantages. I chose dual metric because that helps understand the effect of gravity more intuitively.

How Light Moves in Embedding Diagrams
Since light is massless and travels at c, it doesn't move through time, and only through space (remember the speedometer diagram). Here I visualise how light would move in an embedding diagram, in the absence and presence of mass (see Figure 48 for the initial sketch and Figure 49 for the final illustration).

I show light moving horizontally, just through space, in the two diagrams. As can be imagined, light has to travel a larger distance in the curved spacetime diagram than the one which is flat, in an equal amount of time. This means that the presence of mass, apart from dilating time, also expands space. This has cosmic implications, such as Mercury having an altered orbit- something we'd discuss as we go further.

Visualising space and time curvature together
We know that masses slow down time. We also know that masses increase space. Here, I visualise both of them together using an interactive slider which controls the mass of an object (see Figure 50 for the initial sketch). Two different visualisations demonstrate how time dilation and space expansion increase with increasing mass. Point to remember is that all these are abstract visualizations which help develop intuition, and in no way are visual representations of how curved spacetime literally looks like.

Figure 51 shows the working prototype for the same. The user can control a slider that controls the mass of a spherical object, and can see the effects on its spacetime embedding and space-space diagram in real time.

Light curves in gravitational fields
An observable implication of spacetime curvature is the curving of light. Note that the velocity of light (its speed and direction) are still

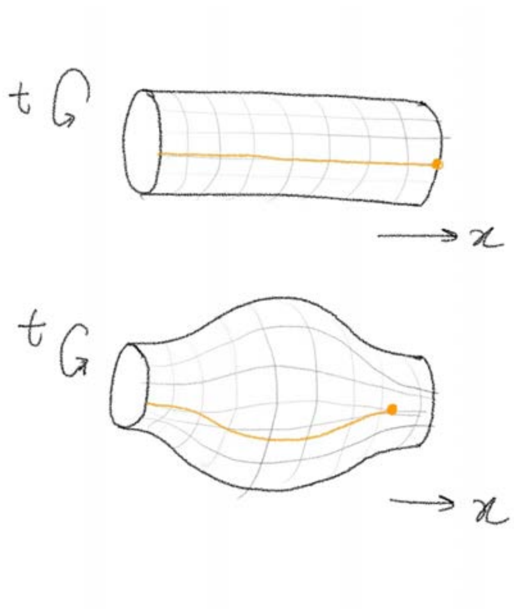

Figure 48: Embedding Diagram showing light: Initial Sketch

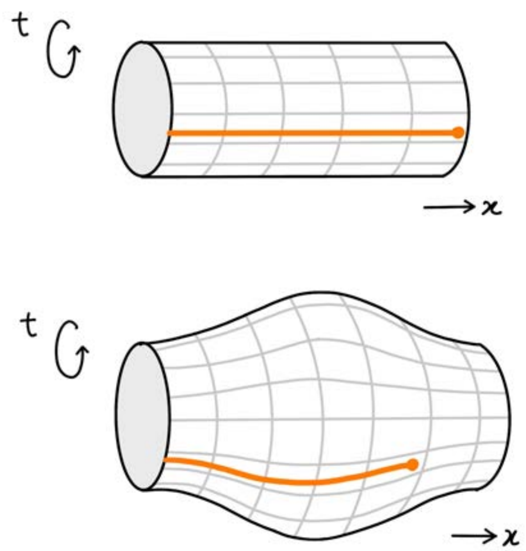

Figure 49: Embedding Diagram showing light

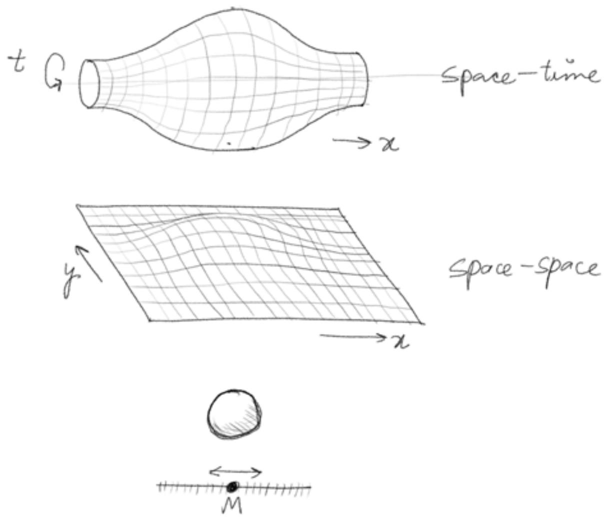

Figure 50: Space and time curvature caused due to mass: Initial Sketch

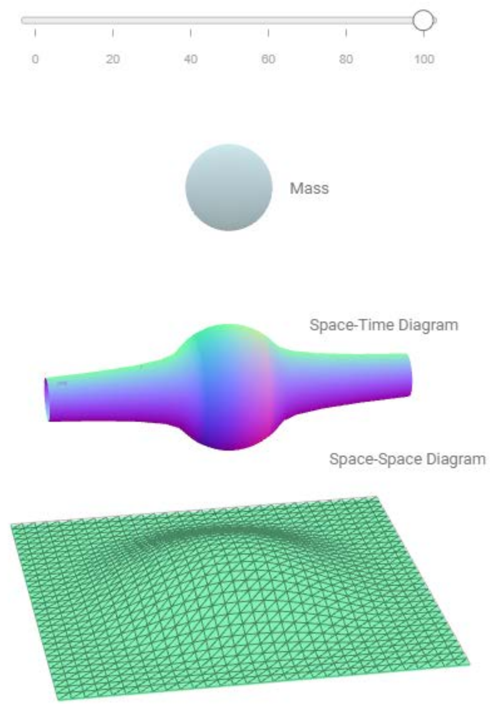

Figure 51: Space and time curvature caused due to mass

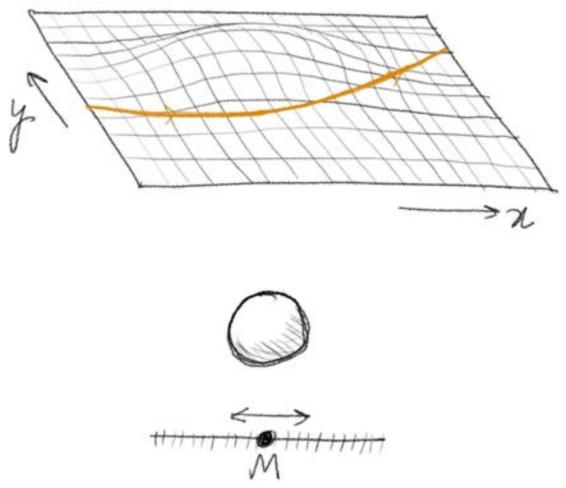

Figure 52: Light curving through distorted spacetime: Initial Sketch

constant. It is the spacetime itself that curves around it, making it look curved to an outside observer.

As light just passes through space and not through time, we can use the space-space diagram above to make a path for light. We can tell intuitively that the curved spacetime causes the beam of light to bend towards the mass (see Figure 52 for the initial sketch).

Figure 53 shows the curvature of light using a 3D model of a space-space diagram. The user can play the visualisation and see the path light traces with an animation.

Visualising all four dimensions of spacetime
4D spacetime is impossible to show or conceive. All we can do is develop abstractions that attempt to demonstrate the way it be-

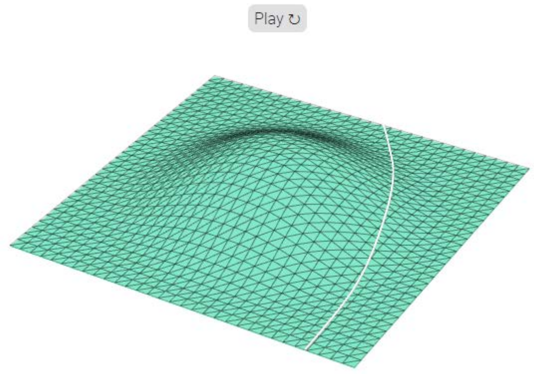

Figure 53: Light curving through distorted spacetime

haves. The job for this task has been largely taken up by the infamous rubber sheet model, which seeks to explain gravity using gravity, and doesn't show time dilation.

However, there are better ways to show how 4D spacetime behaves when a massive object (say, earth) is present in it. Please note that these visualisations are just for developing intuition and conceptual understanding.

The visualisation here (see Figure 54) shows the effect of earth on the space and time around it. Time dilation is shown using a choropleth, where red tends to show slower time, and green shows faster time. The space grid around the earth can be seen to be distorted. The visualisation contains all spatial dimensions, with time as a choropleth.

Discussion on Topic Structuring and Visualisations

My focus in the previous section was to explain how space and time behave differently than one might imagine them to. I, however, have

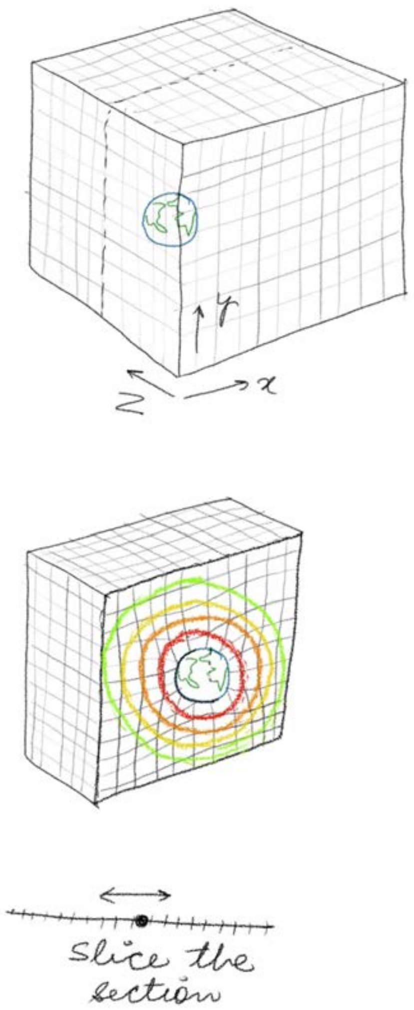

Figure 54: Visualising all four dimensions of spacetime

omitted the following sub-topics which, although are crucial to the topic of relativity, weren't going in with the flow of explaining the behaviour of spacetime:

1. E=mc2 : Crucial to the topic of special relativity, I skipped this topic due to lack of proper visualisations and its inability to fit in the general narrative I was trying to build.
2. Different orbit of Mercury : Presence of the sun results in "extra" space nearer to the sun as compared to further away, which shifts Mercury's orbit by a slight amount. This is what proves general relativity by direct observation.
3. Spectral red shift- An expanding universe : Special relativity shows us how the universe is expanding, because light from the most distant stars is red shifted. [7]

Comparing My Visualisations to Existing Ones

As iterated before, this project aims to consolidate the attempts others have made at visualising relativity and spacetime, and present them in an easy to understand and interactive manner. Following are some of the direct comparisons I have made, of existing information and my intervention.

1. Photon Plank moving through space and time : Mostly this visualisation is shown as either a static image, or as a video. This makes it difficult for the viewer to understand the real-time effects of different velocities on simultaneity. I combat this by providing interactivity that lets the viewer control the speed of the moving cart and see the change in time interval.
2. Rotating disc showing relativistic effects: This was originally visualised by Einstein in his book, Relativity: The Special and the General Theory [4]. I show this visualisation as an animated one, with the speed of rotation being user controlled. This increases immersion and helps the user understand what's happening at different speeds.

3. Embedding diagrams to visualise curved spacetime: Different types of embedding diagrams have been used in scientific resources to visualise the time dilation caused by masses. L. Epstein in his book, Relativity Visualised, gave an introduction to what happens if curved spacetime is rolled around. Dr. Rickard Jonsson further worked on this idea independently, showing different types of embedding diagrams made using mathematical calculations. I use this idea of embedding diagrams to visualise curved spacetime. I further introduce interactivity to show how the embeddings change due to different masses, also demonstrating how the embedding might look like in the presence of a black hole.

Expert Feedback During the Project

I contacted Dr Rickard Jonsson (Astrophysicist at Chalmers University, Sweden) for doubt clearance and some feedback. Dr Jonsson has published two papers specifically for explaining embedding diagrams for curved spacetime. Embedding spacetime via a geodesically equivalent metric of Euclidean signature [11] seeks to explain the Dual Metric embeddings (similar to the ones I use, with a bulge in the center), while Visualising Curved Spacetime [10] explains the Absolute Metric in detail, where time dilation is represented by a crunch instead. Following are the areas I got feedback on:

1. Got basic doubt clearance for how his work compares to L. Epstein's work (which doesn't have a scientific explanation), and how a bulge or a crunch depend on the calculations used to generate the embedding.

2. Got introduced to the Dual Embedding scheme which is a more intuitive yet accurate way of representing spacetime curvature.

3. Got clearance on how proper time is shown in different types of embeddings.

4. Understood how different methods can be used to visualise space crunch caused by the presence of masses, apart from time dilation.

5. Understood how very dense masses collapse to form a black hole, and that is not a smooth and well defined process.

6. Sent him the sketch shown in Figure 55 and got a basic approval on the visual shown. The visual attempts to demonstrate how a spacetime embedding of a black hole might appear according to the Dual Scheme.

Final Prototype

The final prototype consists of the web article that contains static, animated and interactive visuals for explaining the subtopics. It can be accessed here- https://jribh.github.io/Spacetime-Jribh/ (best viewed on Chrome for Laptop/Desktop). Some salient features of the prototype are listed below:

1. A banner on top features an interactive visualisation from the article, and would serve to grab interest of the viewer, as seen in Figure 56. The type of visualisation chosen here was that of a rocket ship experiencing length foreshortening as it flies faster and faster through space. In this particular visualisation, it is easy to see the effects of what is actually happening, while showcasing easily comprehensible visuals like a rocket ship flying through space.

2. Small chunks of text explain each subtopic in a concise manner. Sans-serif font 'Roboto' has been chosen for the paragraphs for a modern look and easy readability.

3. Fixed scroll visualisations accompany the subtopics. The position of the visual would be fixed until the reader reaches the next subtopic. This would ensure that large chunks of text don't dominate the page.

4. Interactive visualisations incorporate sliders to show the changes in real time, as seen in Figure 58. All the interactions are

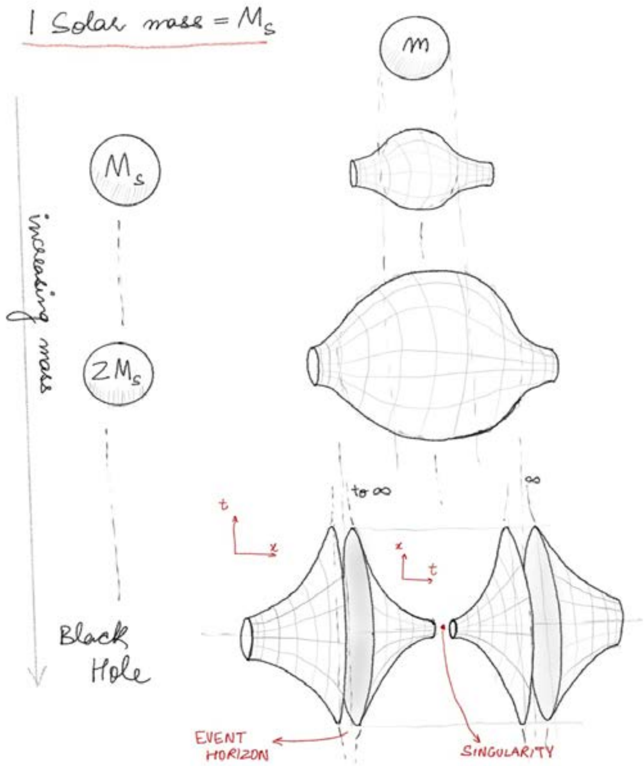

Figure 55: Sketch showing changes in embedding by different masses

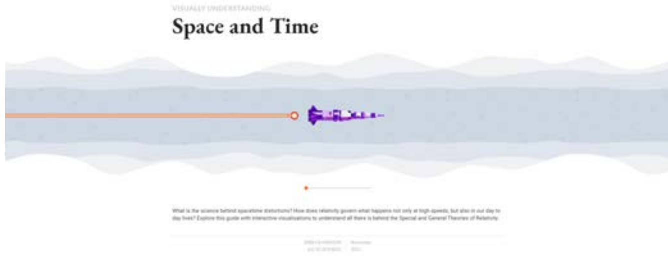

Figure 56: Prototype- Banner

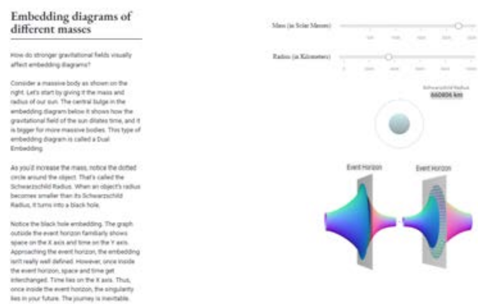

Figure 57: Prototype- 3D Models

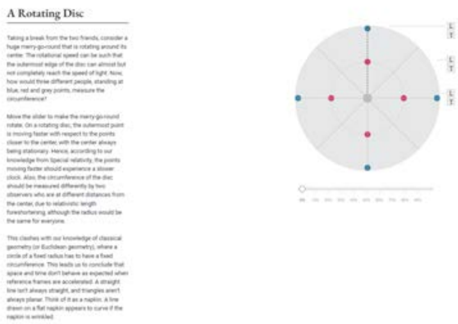

Figure 58: Prototype- Interactive

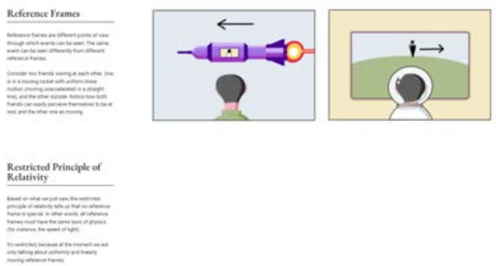

Figure 59: Prototype- Static

made using JavaScript, using majorly two libraries: three.js for the 3D visualisations, and D3.js for the 2D visualisations.

5. Some visualisations incorporate 3D models, that help in visualising curved spacetime with more ease, as seen in Figure 57. This particular visual allows the user to change the mass and radius of a celestial body, and see in real time the effects on its spacetime embedding diagram. The 3D models embedded here, just like all the other 3D models on the page, can be freely rotated around by the audience, which increases immersion and lets the audience view the model from different directions.

6. Not all visualisations are interactive, as that would increase cognitive load on the viewer, as well as increase the loading time of the page. Static visualisations, as seen in Figure 59 explain relatively simpler subtopics. In the image shown here, two static visuals show two different reference frames.

7. The overall layout of the page features text chunks and visualisation boxes with alternating alignments: some subtopics are right aligned while some are left aligned. These are separated by striking text and visuals that serve to categorise the major subtopics, while also making the long web-page less monotonous in terms of layout design.

Product Evaluation Plan

Evaluating the product involves testing its robustness on these major fronts:

1. Testing whether the product is successful in enabling information retention about the topics it covers. This is done using a test that aims to check the knowledge level of participants regarding the topic, after encountering the product. The test would seek to cover the topics that the prototype explains. It is given out as a Google Form to the participants, and can be accessed here: https://forms.gle/5yenJEwjkHcjPU218

2. Qualitative Evaluation: This is geared to measure the motivation amongst users to know more about the covered topics, before and after encountering the product. This makes sure that the audience finds the material insightful, helpful and inspiring. Qualitative feedback is measured by noting down what the audience has to say about the product after a detailed encounter.

3. Expert reviews: Experts such as Dr Rickard Jonsson will review the product based on factual correctness, simplicity of presentation and design approach.

Product Evaluation Results

The product was sent to the participants and the experts, and the following insights were gathered:

1. Information Retention Test: A total of 5 participants got an average score of 6.4 out of 9. Although the score is above 4.5, a less than perfect score can be attributed to the fact that the topics covered were relatively tough to grasp in a short time.

2. Qualitative feedback from participants: A total of 5 participants gave the content an average score of 9.2 out of 10 on the Likert scale, with the scale rating the content from 1 (Not Insightful) to 10 (Very Insightful). Though the participants found the product content rich and the visualisations helpful, most of them struggled to understand the concept of embedding diagrams in one go, the reason probably being the advanced nature of topics which embedding diagrams cover.

3. Expert reviews: The discussions and feedback from Dr. Rickard Jonsson have already been covered in detail. Additionally, at the time of writing this report, I was able to contact some PhD scholars and professors from the detartment of Physics in IIT Bombay. The overall feedback regarding the product was positive, with some concerns about minor issues regarding the page layout on different devices, which I then mended.

Conclusion

It is impossible to visualise or even conceive of how 4 dimensional spacetime truly behaves. We can, however, extract dimensions and develop abstractions that allow us to fit in our observations of the real world as much as possible. Embedding diagrams are one such way of abstraction and representation of something that's not only weirder than you imagine, it's weirder than you can imagine.

The product presented here hasn't been designed with the goal of being quick and easy to comprehend. Instead, I have tried to make sure that it is dense and rich in information, and demands a certain level of mental application from the audience. In return, it aims to simplify and demonstrate a large number of seemingly difficult concepts that come under the Theories of Relativity.

Reflections and Future Scope

The final product is largely in sync with my initial vision, where I aimed to visually explain the concepts of spacetime distortion. There are a few areas I might have changed, and a few areas that can be worked upon further, as discussed below:

1. Since I was largely new to the Javascript libraries like D3 and three.js, I underestimated the time it would take to code out the working prototypes, and in resolving bugs. Although I could achieve most of the working prototypes I had envisioned in the end, I would have appreciated a more coherent UI throughout. Some of the prototypes feel very scientific and bland, and could use a tinge of colour and graphics.

2. Although I tried to have a basic bounding story of two friends in a spaceship ranging across the subtopics, I could have a stronger over-arching theme. That would have increased the connect between various subtopics.

3. I tried to use examples that the audience could relate to, such as merry-go-rounds and light tubes. However, the sheer nature of relativity being active in extreme environments such as speeds close to the speed of light bound me to use some

themes the general public has never experienced in reality, such as rockets and embedding diagrams. In an ideal scenario, examples should be chosen such that people can relate them to their daily lives.

4. The website is currently only accessible for Chrome on Desktop/Laptop. Future work on the product would aim to make it accessible on more devices and browsers.